國立政治大學 101 學年度碩士班招生考試試題

考試科目	統計學	所別	風險管理組 4182	考試時間	2月26日(日) 第三節

(24%) 1. Suppose that X_1 and X_2 are independent, exponential random variables with parameters λ_1 and λ_2 respectively. Please show that

(6%) (1) $X_3 = \min(X_1, X_2)$ is independent of the event $\{X_1 < X_2\}$, and

(6%) (2) $P(X_3 = X_1) = P(X_1 < X_2)$.

Furthermore, please find:

(6%) (3) The distribution functions of $U = \max(X_1 - X_2, 0)$ and

$V = \max(X_1, X_2) - \min(X_1, X_2)$, and

(6%) (4) $P(X_1 \le s < X_1 + X_2)$ where $s > 0$.

(16%) 2. Please show that $P(\cap_{i=1}^{m} A_i) \ge 1 - \sum_{i=1}^{m} P(A_i^C)$ for any events $A_1, A_2, \ldots A_m$.

(20%) 3. Please show that $Var(Y) = E(Var(Y|X)) + Var(E(Y|X))$.

(16%) 4. Suppose that you face a rabbit race bet. There are n rabbits in total and the dealer offered odds $f(k)$ for the k-th rabbit where $\sum_{k=1}^{n} (f(k) + 1)^{-1} < 1$. Please specify a sure-win strategy in this bet.

(24%)5. Let $Z_1, \ldots, Z_n$ be i. i. d. random samples from $f(z|\theta) = \theta z^{\theta-1}$, $0 \le z \le 1$, $0 < \theta < \infty$.

(8%) (1) Please find the MLE of θ.

(8%) (2) Is the MLE a consistent estimator?

(8%) (3) Please find of MM estimator of θ.

備註	試題隨卷繳交