

考試科目	微積分	所別	4183 風險管理學系 (精算組)	考試時間	2月6日(日)第一節
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1. (20 分) Suppose f and g are two functions with derivatives.
Prove that

$$\frac{d}{dt}(f(t)g(t)) = \frac{d}{dt}f(t) \cdot g(t) + f(t) \cdot \frac{d}{dt}g(t)$$

2. (30 分)

- (a) Let n be a positive integer, compute

$$\lim_{\theta \rightarrow 0} \frac{1 - \cos(n\theta)}{\theta^2}$$

- (b) Let $x > 1/3$ be a constant and $\log(\cdot)$ denote the natural logarithm, compute

$$\lim_{\theta \rightarrow 0} \frac{\log(3x + 5\theta) - \log(3x)}{\theta}$$

- (c) Compute

$$\lim_{\theta \rightarrow 0^+} \theta^{3\theta}$$

3. (20 分) Compute

- (a)

$$\int_0^1 \frac{1}{\sqrt{x}} dx$$

- (b)

$$\int_0^1 \frac{1}{x\sqrt{x}} dx$$

4. (30 分) Let $f(t) = \exp(-t^2/2)$ and θ is a real number. Compute

- (a)

$$A = \int_0^{\infty} f(t) dt$$

- (b)

$$g(\theta) = \int_{-\infty}^{\infty} \exp(\theta t) f(t) dt$$

- (c)

$$\frac{d^2}{d\theta^2} g(\theta) |_{\theta=0}$$