

招生學年度	100	招生類別	碩士班
系所班別	材料科學與工程學系碩士班、光電工程學系碩士班(甲組、乙組)		
科目	工程數學		
注意事項	本考科可使用掌上型計算機		

1. (10%) Solve $(2x + y^2)dx + xydy = 0$

2. (10%) Determine whether the following sets of vectors are linearly dependent or independent. Why? (a) $\{(1,1,2), (1,2,1), (3,4,5)\}$ in \mathbb{R}^3 ; (b) $\{(0,1), (1,4), (2,3)\}$ in \mathbb{R}^2 .

3. (10%) For a matrix $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, find the eigenvalues and the corresponding orthonormal eigenvectors.

4. (10%) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = 4x^2y$, $y(0) = 2$, $y'(0) = 1$, Please estimate the value of $y(0.1)$ with at least 5 significant digits.

5. (15%) Find the Fourier transform of $f(x) = \begin{cases} xe^{-x}; & x > 0 \\ 0; & x < 0 \end{cases}$

6. (15%) Solve $\frac{d^2y}{dx^2} + y = \tan^2x$

7. (15%) $t\frac{d^2y}{dx^2} + \frac{dy}{dx} + ty = 0$, $y(0)=1$, $y'(0)=0$. Find the Laplace transform of $y(t)$.

8. (15%) The 3-D Schrödinger equation is

$$\nabla^2\psi(x,y,z) + \frac{2m}{\hbar^2}(E - V(r))\psi(x,y,z) = 0, \text{ where } \nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}. \text{ Please}$$

re-write this equation in spherical coordinate (r, θ, ϕ) .

