招生學年度	100	招生類別	碩士班
系所班別	應用數學系 統計碩士班		
科目	機率與統計		
注意事項	本考科禁止使用掌上型計算機；含機率論與統計學		

1. (10 points) State and Prove the Tchebichev's inequality.

2. (10 points) Let the *j.p.d.f.* of X, Y and Z be

$$f(x,y,z)=\frac{6}{(1+x+y+z)^4},\quad \text{if } x>0, y>0, z>0,$$

and 0, otherwise. Let $T=X+Y$.
Determine the conditional *p.d.f.* of X given $T=t$, for any $t>0$.

3. Let X and Y be independent $N(0,1)$ random variables and $\lambda\in R$ a given constant. Define a new random variable T by

$$T=\begin{cases} Y & \text{if } X<\lambda Y,\\ -Y & \text{otherwise.}\end{cases}$$

   (a) (10 points) Derive the p.d.f. of T.

   (b) (10 points) Calculate $E(T)$ and $Var(T)$.

4. (15 points) Suppose that the family of p.d.f.'s of the statistic T, $\{g(t;\theta):\theta\in\Omega\}$, has MLR (monotone likelihood ratio) in t.
Show that for any given number c, if $\theta_1<\theta_2$ then $P_{\theta_1}(T>c)\le P_{\theta_2}(T>c)$, that is $P_\theta(T>c)$ is a non-decreasing function of θ.

5. (15 points) Let $X_1,\ldots,X_n,\ldots$ be *i.i.d.* as $U[0,\theta]$, let $X_{(n)}=\max\{X_1,\ldots,X_n\}$, the MLE (maximum likelihood estimator) of θ, determine the limiting distribution of $n[\theta-X_{(n)}]$.

6. (15 points) Let $X_1,\ldots,X_n$ be *i.i.d.* $N(\mu,\sigma^2)$ random variables, where $\sigma>0$ is known. Find the UMVUE (uniformly minimum variance unbiased estimator) of μ^2, and investigate whether the Cramér-Rao lower bound is attained.

7. (15 points) One observation is taken on a discrete random variable X with p.d.f. $f(x;\theta)$, where $\theta\in\Omega=\{\theta_0,\theta_1,\theta_2,\theta_3\}$.

Values of $f(x;\theta)$

x	2	3	4	5	6	7	8	9	10	11	12
θ_0	.01	.01	.01	.01	.01	.01	.01	.01	.01	.01	.90
θ_1	.01	.009	.008	.007	.006	.005	.006	.007	.008	.009	.925
θ_2	.20	.10	.09	.08	.07	.06	.05	.05	.05	.05	.20
θ_3	.30	.09	.09	.08	.08	.07	.07	.06	.06	.05	.05

Derive a level $\alpha=0.05$ LRT (likelihood ratio test) for testing
$H_0:\theta\in\{\theta_0,\theta_1\}$ $v.s.$ $H_1:\theta\notin\{\theta_0,\theta_1\}$.
Is the test you obtained a UMP level 0.05 test?