

1. Consider a IS-LM graph for a closed-economy where the real interest rate is on the vertical axis. Consider also a IS\*-LM\* graph for an open-economy (the Mundell-Fleming model) where the exchange rate ( $e=(\text{foreign dollar})/(\text{domestic dollar})$ ) is on the vertical axis. Now assume there is an increase in tax ( $T$ ).
- (a) (10pt) Draw an IS-LM graph showing the effect of an increase in  $T$  on a closed economy. Draw two IS\*-LM\* graphs showing the effect on an open economy with fixed exchange rates and floating exchange rates, respectively. Explain your graphs.
- (b) (10pt) In which of the three scenarios (closed, open with fixed exchange rates, open with floating exchange rates) does the increase in  $T$  has the largest effect on output? Which scenario has the least effect on output? What is the economic reason? Your explanation determines the grade.
2. Consider a two-period consumption choice model. Assume an agent is endowed with non-labor income  $y_1$  and  $y_2$  in period 1 and 2, respectively. The agent's life-time utility is

$$U = u(C_1) + \beta E_t[u(C_2)],$$

where  $u(\cdot)$  is the instantaneous utility function,  $C_i$  is the consumption in period  $i$ , and  $0 < \beta < 1$  is the preference discount factor. In period 1, the income can be used either to consume or to buy  $S$  units of bonds which costs  $P_1$  per unit. The bond will mature in period 2 and each unit will have the payoff  $X_2$  which is stochastic. Payoff of the bond can be used for consumption in period 2. The bond has no value left after period 2 and the agent dies after period 2. The agent's task is to maximize the life-time utility by choosing the optimal amount of  $S$ .

- (a) (5pt) Write down the agent's budget constraint of period 1.
- (b) (5pt) Write down the agent's budget constraint of period 2.
- (c) (5pt) Write down the agent's utility maximization problem.
- (d) (7pt) Derive the problem's first-order condition and show the agent's willingness to pay for one unit of the bond (i.e., solve for  $P_1$ ).
- (e) (8pt) Suppose for some reason that the positive correlation between  $u'(C_2)$  and  $X_2$  suddenly increases. Does the agent's willingness to pay for the bond ( $P_t$ ) goes up or down after the increase in the correlation? WHY?
3. Consider a Solow model with technological progress and population growth. A final good in the economy is produced according to:  $Y_t = (A_t N_t)^{(1-\alpha)} K_t^\alpha$ ,  $0 < \alpha < 1$ , where  $A_t$  is the technology level,  $K_t$  is the predetermined capital stock and  $N_t$  is the population number. The capital stock evolves according to  $K_{t+1} = I_t + (1 - \delta)K_t$ , where  $0 < \delta < 1$ ,  $K_0$  is given, and  $I_t$  is investment.  $A_t$  and  $N_t$  evolve according to  $A_t = A_{t-1}\gamma_a$ , and  $N_t = N_{t-1}\gamma_n$  where the value of  $A_0$  and  $N_0$  are given and  $\gamma_a \geq 1$  and  $\gamma_n \geq 1$  are the growth rate of technology and population respectively. The resource constraint is  $Y_t = C_t + I_t$  where  $C_t$  is consumption in period  $t$ . Suppose the economic agent saves  $s$  fraction of their production each period. Therefore  $I_t = sY_t$

- (a) (6pt) Define the capital stock per effective unit of labor,  $k_t = \frac{K_t}{A_t N_t}$  and derive a non-linear difference equation in a single state variable,

$$\gamma_a \gamma_n k_{t+1} = s k_t^\alpha + (1 - \delta) k_t$$

- (b) (6pt) Solve for the steady state level of capital per effective worker,  $\frac{K_t}{A_t N_t}$ , output per effective worker,  $\frac{Y_t}{A_t N_t}$ , and saving per effective worker,  $\frac{S_t}{A_t N_t}$ .
- (c) (6pt) What are the growth rates of output,  $Y$ , consumption,  $C$ , and investment,  $I$  respectively?
- (d) (6pt) Use a diagram to determine the steady state quantity of capital per effective worker.
- (e) (6pt) Suppose that the economy is initially in a steady state and that some of the national's capital stock is destroyed because of a natural disaster. Will the economy converge back to the same steady state? Use a diagram to show the dynamic adjustment of capital per worker over time.

4. Consider a static economy where a household lives only for one period. The representative household chooses between consumption,  $c$ , and leisure,  $l$  to maximize his or her utility. Suppose that the household has  $h$  hours of time that he can choose between working,  $n$ , and leisure,  $l$ , i.e.,  $h = l + n$ . The household receive wage income  $wn$ , which is denoted as units of consumption goods, for his labor service. In addition, the preference of the household can be expressed as  $U(c, l) = \log c + \theta \frac{l^2}{2}$ , where  $\theta$  is parameter determine the relative weight from utility of leisure. So, we can restate the household's optimization problem as

$$\begin{aligned} \max_{c, l} \quad & \log c + \theta \frac{l^2}{2} \\ \text{subject to} \quad & \\ & c = w(h - l) \\ & c \geq 0 \\ & h \geq l \geq 0. \end{aligned}$$

- (a) (6pt) Draw the household's budget constraint and show his or her optimal choice of consumption and leisure. Please provide the economic interpretation for the figure.
- (b) (6pt) Please solve for the optimal choice of leisure,  $l$ . In particular, express  $l$  as a function of real wage,  $w$ , the parameter  $\theta$  and the time endowment,  $h$ .
- (c) (8pt) Suppose that a household can earn a higher wage rate for working overtime. That is, for the first  $q$  hours the household works, he or she can receive a real wage rate of  $w_1$ , and hours worked more than  $q$ , he or she can receive  $w_2$ , where  $w_2 > w_1$ . Draw the new budget constraints and show the optimal choice for a household. Explain the intuition why a household would never work  $q$  hours or anything very close to  $q$  hours.