國立政治大學　106 學年度 碩士班 招生考試試題

考試科目	數理統計學 4414	系所別	統計學系	考試時間	2 月 18 日(六) 第三節

1. Let Y follow a truncated normal distribution with pdf

$$f(y)=\frac{\phi(y)}{1-\Phi(a)},\quad y>a,$$

where $\phi(y)$ and $\Phi(a)$ are respectively the pdf and cdf of N(0,1).

Find $\mu=E(Y)$ and $\sigma^2=V(Y)$. (10%)

2. Let $(X,Y)\sim f(x,y)=\exp(-x),\ x>y>0.$

1) Find $Cov(X,Y)$. (6%)

2) Find the joint pdf of $U=X$ and $V=X+Y$. (6%)

3) Find the marginal pdf of U, and the marginal pdf of V. (8%)

4) Find $P[U\le 1.5\,|\,V=2]$. (6%)

3. Let $X_1,\ldots,X_n$ be a random sample from $f(x;p)=p(1-p)^{x-1}$, $x=1,2,\ldots,$ with $\mu=1/p$ and $\sigma^2=(1-p)/p^2$.

1) Find the MLE for $P[X>m]=(1-p)^m$, for some $m=1,2,\ldots$ (6%)

2) Find the limiting distribution for $\sqrt{n}(1/\bar{X}_n-p)$. (6%)

3) Find the UMVUE for $P[X=1]=p$. (12%)

4. For a simple linear regression model with no intercept:

$Y_i=\beta x_i+\varepsilon_i$, $i=1,\cdots,n$, and $\varepsilon_i\sim iid\quad N(0,\sigma^2)$.

1) Assume that σ^2 is known.

1a) Find MLE of β, *i.e.* $\hat{\beta}$, and specify the distribution of $\hat{\beta}$. (10%)

1b) Derive the UMP size α test of $H_o:\beta=0$ vs. $H_a:\beta>0$. (10%)

1c) A test rejects H_o if $\dfrac{\hat{\beta}}{\sigma/\sqrt{\sum_{i=1}^n x_i^2}}>z_\alpha$. Find the power for it at $\beta=1$. (6%)

2) Assume that β and σ^2 are both unknown.

Derive the likelihood ratio test for $H_o:\beta=0$ vs. $H_a:\beta\neq 0$. (14%)

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。