國立政治大學　106　學年度 碩士班 招生考試試題

考試科目	基礎數學 41412	系所別	統計學系	考試時間	2 月 18 日(六) 第一節

Note. You need to show your work in your solutions for the following problems instead of giving final answers only.

1. (20 points) Suppose that

$$f(x) = \begin{cases} [redacted] & \text{if } x = 0; \\ x^{-3} & \text{if } x < 1 \text{ and } x \neq 0; \\ -1 + x & \text{if } 1 \leq x < 2; \\ xe^{-x^2} & \text{if } x \geq 2. \end{cases}$$

   Find $\int_1^\infty f(x)dx$ and $\int_{-\infty}^\infty f(x)dx$.

2. (16 points) Find the following limits.

   (a) $\lim_{x\to\infty} \left(1 + \frac{2}{x}\right)^x$.

   (b) $\lim_{x\to\infty} \frac{\int_x^\infty e^{-t^2}dt}{xe^{-x^2}}$.

   (c) $\lim_{x\to\infty} \frac{x + \cos(x)}{x - \sin(x)}$.

   (d) $\lim_{n\to\infty} \frac{1}{n}\sum_{k=1}^{n} \frac{k}{n}\sin\left(\frac{2\pi k}{n}\right)$.

3. (14 points) Suppose that u is a differentiable function of x such that

$$u + x\cos(u) = 1$$

   and $u = 0$ when $x = 1$. Let

$$f(x, y) = \int_0^x \int_y^{y+s} s(t - y)dtds + (u + y)^2 - \frac{x}{2}$$

   for x, $y \in (-\infty, \infty)$. Determine whether f has a local minimum or a local maximum at the point $(1, 0)$. Justify your answer.

4. (12 points) Suppose that a is a real number and $a \neq 0$. Let

$$A = \begin{pmatrix} 1 & a & 0 \\ a & 1 & a \\ 0 & a & 1 \end{pmatrix}.$$

   (a) Show that 1 is an eigenvalue of A.

   (b) Find an eigenvector of A associated with the eigenvalue 1.

   (c) Find all eigenvalues of A that are not equal to 1.

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

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5. (24 points) Suppose that n is a positive integer and let

$V_n = \{p : p \text{ is a polynomial of real coefficients on } [0,1] \text{ of degree at most } n.\}$.

Then it is clear that V_n is a linear space, where the vector addition is the usual addition for polynomials and a vector multiplied by a scalar means a polynomial multiplied by a real constant. Let

$$W = \left\{p \in V_n : \int_0^1 p(x)dx = 0\right\}$$

and

$$W^* = \left\{q \in V_n : \int_0^1 q(x)p(x)dx = 0 \text{ for every } p \in W\right\}.$$

(a) Show that W and W^* are linear spaces.
(b) Find the dimension of W.
(c) Find the dimension of W^* and give a set of basis for W^*.

6. (14 points) Suppose that A is a 3×3 matrix of the following form

$$\begin{pmatrix} 1 & B \\ O & D^{-1} \end{pmatrix},$$

where $B = \begin{pmatrix} 2 & 3 \end{pmatrix}$ is a 1×2 matrix, O is a 2×1 matrix of zeros, and D^{-1} is the inverse matrix of a 2×2 matrix D. Express the inverse matrix of A in terms of D.

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