

考試科目	計算機數學 8/4/3	所別	資訊科學系	考試時間	2月28日(日)第三節
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- I. Discrete Mathematics (60%)
- (10%) Consider that  $2^n > n^3$  for  $n \geq K$ , where  $K$  is a positive integer. What is the smallest value of  $K$ ?
  - (10%) If we want to find the sum of 11 numbers by using the hypothetical computer that has an instruction which computes the sum of three numbers, how many times the addition instruction will always be executed? [Hint: Any sequence of execution of this instruction to obtain the result is a regular ternary tree with nine leaves.]
  - (10%) Consider the big-O notation and the following two functions,  $g$  and  $h$ . What is  $gh$ ?  

$$g = n + O\left(\frac{1}{n}\right), h = n^{1/2} + O\left(\frac{1}{n^{1/2}}\right)$$
  - (10%) Simplify the following proposition.  

$$(a \wedge b \wedge c) \vee (\sim a \wedge b \wedge c) \vee (a \wedge b \wedge \sim c) \vee (\sim a \wedge b \wedge \sim c) \vee (a \wedge \sim b \wedge \sim c)$$
  - (10%) Consider the following recursive function. What is  $f(n)$ , or how is  $f(n)$  defined directly?  

$$f(0) = 2, f(1) = 6, f(n) = 4f(n-1) - 4f(n-2) \text{ for } n > 1$$
  - (10%) Consider a connected graph that can be described by the following adjacency matrix  $A$  and degree matrix  $D$ . What is the number of spanning trees of the graph?  

$$A = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}, D = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$
- II. Linear Algebra(40%)
- (10%) Please find the inverse of each of the following matrices.  
 (1)  $A = \begin{bmatrix} 2 & 3 \\ 5 & 8 \end{bmatrix}$       (2)  $A = \begin{bmatrix} 2 & 4 \\ 5 & 10 \end{bmatrix}$
  - (10%) Suppose that  $A = \begin{bmatrix} 3 & 4 & 2 \\ 2 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$  and  $R = [1 \ 3 \ 2]$ . We construct a new matrix  $B$  by adding  $R$  to the third row of  $A$ . We construct a new matrix  $M$  by replacing the third row of  $A$  by  $R$ . Please find the determinants of  $A$ ,  $B$ , and  $M$ .
  - (10%) Suppose that  $A = \begin{bmatrix} 0 & 1 & -1 & 1 \\ 1 & 2 & -1 & 2 \\ 0 & 3 & -3 & 3 \end{bmatrix}$ . Please find the determinantal rank of  $A$ .
  - (10%) Suppose that  $A = \begin{bmatrix} 5 & -1 & 2 \\ 0 & 2 & 0 \\ 8 & 4 & 5 \end{bmatrix}$ . Please find the eigenvalues of  $A$  and the corresponding eigenvectors.

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。
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