考試科目	基礎數學	所別	統計學系 4141	考試時間	2 月 26 日(日) 第一 節

1. (12 points) Let A be a nonsingular $n \times n$ matrix. Which of the following statements are correct? If a statement is not correct, say how to correct it.
   (a) There exists an $n \times n$ matrix B such that $AB = I_n$, where I_n is the identity matrix.
   (b) For any vector $b \in R^n$, the linear system $Ax = b$ has a unique solution.
   (c) A has rank 0.
   (d) The dimension of the column space of A is less than n.

2. (9 points) Let A be an $m \times n$ matrix with rank $A = n$. Which of the following statements are correct? If a statement is not correct, say how to correct it.
   (a) $A^T A$ is nonsingular.
   (b) $A^T A$ is positive definite.
   (c) Let b be any vector in R^n. The linear system $Ax = b$ may have multiple solutions.

3. (20 points) Let A and B be $n \times n$ matrices such that $AB = I_n$. Suppose that A has n eigenvalues $\lambda_1, ..., \lambda_n$. Answer the following questions.
   (a) Write down the characteristic polynomial of A.
   (b) Find the eigenvalues of B.
   (c) Find the eigenvalues of A^T, the transpose of A.
   (d) Express the trace of A in terms of λ_i.

4. (9 points) Let

$$A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}.$$

   (a) (3 points) Find the eigenvalues of A.
   (b) (6 points) Diagonalize the matrix A.

備註	試題隨卷繳交

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5.

(A) Evaluate $\int_{-\infty}^{\infty} e^{-x^2}\,dx$. (5%)

(B) Evaluate $\int_{1}^{\sqrt{3}} \arctan\left(\frac{1}{x}\right)dx$. (5%)

6.

Suppose $f\left(\frac{x^2-1}{x^2+1}\right)=x, x>0$. Find $f'(0)$. (10%)

7.

Find $\frac{d}{dx}\cos x\int_{\tan x}^{0} \tan t\,dt$ (10%)

8.

(A) Evaluate $\lim_{n\to\infty}\left(1+\frac{0.7}{n^2}\right)^{5n}$. (5%)

(B) Evaluate $\lim_{n\to\infty}\frac{1}{n}\left(\sqrt{\frac{1}{n}}+\sqrt{\frac{2}{n}}+\cdots+\sqrt{\frac{n}{n}}\right)$. (5%)

9. Test the series $\sum_{n=0}^{\infty}(-1)^n\frac{(x-3)^n}{2n+1}$ for convergence or divergence. In addition, find the interval of convergence if the series converges. (10%)

備註	試題隨卷繳交