PART I. Multiple Choice Questions (單選題) 60%，3 points each.
INSTRUCTIONS :You do not need to show the detailed steps (無需列出計算過程).

1. Which of the following is mostly correct:
   (a). If event A and B are independent, then A and B are mutually exclusive
   (b). If event A and B are mutually exclusive, then A and B are not independent
   (c). If the covariance of random variables *X* and *Y* equals to zero, then X and *Y* must be independent.
   (d). The relationship between random variables *X* and *Y* is that $Y = a + bX$， where a and b are constants, then the correlation coefficient of *X* and *Y* must equal to 1.

2. Firm ABC is concerned about a take over attempt by firm XYZ. Firm ABC estimates that the probability of their stock dropping below its book value of $50 is 0.7. The probability of a take over attempt by XYZ is 0.6. Given the stock price of ABC goes below 50, the probability of a takeover attempt by XYZ is 0.8. What is the probability that XYZ does not try to take over ABC, given that ABC stock does not drop below 50?
   (a). 0.8667 (b). 0. 6667 (c). 0. 65 (d). 0.44

3. The probability density function of random variable *X* is given by $f(x) = 6x(1-x), 0 \le x \le 1$. Then, the median of *X* is
   (a). 1/6. (b). 1/3. (c). 1/2. (d). 2/3.

4. Random variable X denotes the number of customers arriving at a bank. We also know that $E(X) = 100$ and $Var(X) = 10$, Using Chebyshev's Inequality to find the upper bound on the probability that the number of customers is outside the range [75, 125], which is
   (a). 2/125. (b). 4/125. (c). 123/125. (d). 121/125.

5. The joint distribution of continuous random variables *X* and Y is $f(x,y) = 2x, 0 \le x \le 1, \quad 0 \le y \le 1$, Then, the covariance of *X* and *Y* is
   (a). 1/6. (b). 1/3. (c). 1/2. (d). 2/3.

6. Suppose we cut a horizon line, which is 5 feet long, into two pieces randomly. Let *X* be the random variable, which represents the length of piece on left hand side. Correspondingly, 5-*X* is the length of piece on right hand side. Find the expectation of the cross product of the length of two pieces, i.e., $E[X(5-X)]$ which is: (feet^2)
   (a). 5/2 (b). 25/2 (c). 25/4 (d). 25/6

7. There are 400 identical machines work independently. The probability of failure on each machine is 0.02 during the given time period. Use Poisson approximation to estimate the probability that **more than one** machine are fail.
   (a). e^{-8} (b). $1-e^{-8}$ (c). $1-9e^{-8}$ (d). $9e^{-8}$

8. Suppose X follows an exponential distribution ($f(x)=\frac{1}{\mu}e^{-x/\mu},\ 0<x<\infty$)，and $P(x\le 1)=P(x>1)$，then Var(x) is
(a). 2*ln*(2) (b). *ln*(2) (c). e^{-4} (d). e^{-2}

9. Let $X_1,X_2,....,X_n,X_{n+1},....,X_{m+n}$, be random samples from a population with distribution ~ $N(0,\sigma^2)$. Let $Y=\frac{\sqrt{m}\sum_{i=1}^{n}X_i}{\sqrt{n}\sqrt{\sum_{i=n+1}^{n+m}X_i^2}}$ 。Then Y has;
(a). a t distribution with degree of freedom $df=n$.
(b). a t distribution with degree of freedom $df=n-1$.
(c). a t distribution with degree of freedom $df=m$.
(d). a t distribution with degree of freedom $df=m-1$.

10. Let $X_1,X_2,....,X_n,X_{n+1},....,X_{m+n}$, be random samples from a population with distribution ~ $N(0,\sigma^2)$. Let $Y=\frac{m\sum_{i=1}^{n}X_i^2}{n\sum_{i=n+1}^{n+m}X_i^2}$ 。Then; Y has
(a). a F distribution with degree of freedom $df_1=n-1,\quad df_2=m-1$
(b). a F distribution with degree of freedom $df_1=n,\quad df_2=m$
(c). a F distribution with degree of freedom $df_1=m-1,\quad df_2=n-1$
(d). a F distribution with degree of freedom $df_1=m,\quad df_2=n$

11. Survey the income from 25 persons and find their sample mean is 35 (thousand). Suppose its distribution is unknown but the population standard deviation of a person's income is known to be 6. Use Chebyshev inequality to construct its 96% confidence interval for average income, which is
(a). [32, 38] (b). [30, 40] (c). [29, 41] (d). [28, 42]

12. The *p.d.f.* of random variable X is $f(x)=\frac{1}{\mu}e^{-x/\mu},0<x<\infty$. To test the hypothesis $H_0:\mu=10\ vs.\ H_a:\mu=20$. Let $X>15$ be the rejection region. Then, the probability of Type I error (α) is
(a). $1-e^{-3/2}$ (b). $e^{-3/2}$ (c). $1-e^{-3/4}$ (d). $e^{-3/4}$

13. Following question 12, the testing power ($1-\beta$) is.
(a). $1-e^{-3/2}$ (b). $e^{-3/2}$ (c). $1-e^{-3/4}$ (d). $e^{-3/4}$

14. The *p.d.f.* of random variable X is $f(x)=(\alpha+1)x^{\alpha},0<x<1$. Then, the method of moment estimator (MOM) for parameter α is (Hint: let $E(X)=\overline{X}$):
(a). $\frac{1-2\overline{X}}{\overline{X}-1}$ (b). $\frac{2\overline{X}-1}{\overline{X}-1}$ (c). $\frac{\overline{X}-1}{2\overline{X}-1}$ (d). $\frac{1-\overline{X}}{2\overline{X}-1}$

15. Which of the following statements about the analysis of variance (ANOVA) is incorrect?
(a). Under the assumption of common variance, mean square for error is an unbiased estimator for the variance
(b). Under the assumption of common variance, mean square for treatment may not be an unbiased estimator for the variance
(c). Using *t* statistic to test the equivalence of paired treatment mean, given the significant level, may increase the actual type I error.
(d). To test the equivalence of treatment mean, using randomized block design is always more powerful than completely randomized design.

16. Consider a multiple regression with two independent variables. If we find the correlation coefficient between the two independent variables approaches to one, then
(a). the variance inflation factor will approach to zero
(b). the variance inflation factor will approach to infinity
(c). the standard deviation of the slopes will approach zero
(d). the standard deviation of the slopes will approach infinity

17. Consider a simple regression with sample size *n*=40, you are told that $\sum y_i = 160$，$\sum x_i = 80$ and the Least Square estimate of β_0 (intercept) is 2, then β_1 (slope)= ?
(a). 1/2 (b). 1 (c). 2 (d). 4

18. Which of the following is false? The Least Square estimator for regression
(a). is a linear combination of observation *y*
(b) is consistent only if independent variables and error term are independent
(c) can not be derived if the independent variables are linearly dependent
(d) can not be derived when Gauss Markov assumptions do not hold

19. When fitting the regression model $Y = \alpha + \beta X + \varepsilon$, you find that the error variance is proportional to $X^{-1/2}$. Which of the following models can be used to correct the above form of heteroskedasticity?
(a). $YX^{1/4} = \alpha X^{1/4} + \beta X^{5/4} + \varepsilon^*$
(b). $YX^{1/4} = \alpha + \beta X^{5/4} + \varepsilon^*$
(c). $YX^{1/2} = \alpha X^{1/2} + \beta X^{3/2} + \varepsilon^*$
(d). $YX^{-1/4} = \alpha X^{-1/4} + \beta X^{3/4} + \varepsilon^*$
(e). $YX^{-1/2} = \alpha X^{-1/2} + \beta X^{1/2} + \varepsilon^*$

20. A financial manager uses firm's operating profit income from previous year as the dependent variable and firm's leverage in the current year as the independent variable in a simple regression. Using 20 years of data, the manager estimates the model slope coefficient with the least square estimator $\hat{\beta}$, but does not take into account that the error term in the model exhibit to be first order autoregressive with autocorrelation coefficient $\rho > 0$. Which of the following statements is false?

(a). The R^2 probably gives an overly optimistic picture of the success of the regression.
(b). The estimator of the standard deviation of $\hat{\beta}$ is biased downward.
(c). The estimator $\hat{\beta}$ is inconsistent.
(d). The estimator $\hat{\beta}$ is inefficient.

PART II.

INSTRUCTIONS : There are three questions worth a total of 40 points in Part II. It is necessary to show the detail solution processes on your answer sheet. (必須詳列其計算過程，只寫答案不予計分)

1. Suppose that financial crisis occurs in accordance with the assumption of the Poisson probability distribution at a rate of 1 times per five years.
   (1). Fine the probability that at least twice financial crises during next decay.
   (2). Find the probability distribution of the time between now and the next earthquake. (16%)

2. Let P be the probability of getting a head when a given coin is tossed. In order to test $H_0: p = 0.5$ vs. $H_1: p \neq 0.5$, a trail, tossing the coin 5 times, is conducted, and let X be the number of head obtained. If a test rejects H_0 when $X = 0$ and $X = 5$, answer the following questions.
   (1) Find the probability of type I error
   (2) If $p = 0.6$, find the probability of type II error.
   (3) If $p = 0.6$, find the power of test.
   (4) If the trial obtains 4 head, find the p-value of this test. (16%)

3. An economist wishes to study the monthly trend in the Dow Jones Industrial Average (DJIA). Data collected over the past 40 months were used to fit the model, where y=monthly close of the DJIA and t=month (1,2,…,40). The regression results appear below:

   $\hat{y}_t = 28 + 0.15 y_{t-1}$ $R^2 = 0.66$ $MSE = 144$ $t_{\text{int}} = 1.72$ $t_{y_{t-1}} = 3.76$ $DW = 1.32$

   Conduct a test to determine if the residuals are positively correlated (you need to state hypotheses, level of significant, decision rules,…..etc) (8%)

附表 3. Table of normal distribution

Z Table

Entries in the body of the table represents areas under the curve between $-\infty$ and z

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

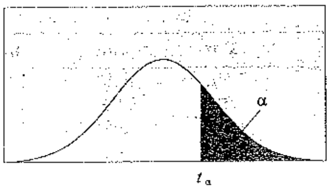
附表 1. Durbin-Watson critical values

X variables, excluding the intercept

Observations		1		2		3		4		5	
N	Prob.	D-L	D-U	D-L	D-U	D-L	D-U	D-L	D-U	D-L	D-U
15	0.05	1.08	1.36	0.95	1.54	0.82	1.75	0.69	1.97	0.56	2.21
	0.01	0.81	1.07	0.7	1.25	0.59	1.46	0.49	1.70	0.39	1.96
20	0.05	1.20	1.41	1.10	1.54	1.00	1.68	0.90	1.83	0.79	1.99
	0.01	0.95	1.15	0.86	1.27	0.77	1.41	0.68	1.57	0.60	1.74
25	0.05	1.29	1.45	1.21	1.55	1.12	1.66	1.04	1.77	0.95	1.89
	0.01	1.05	1.21	0.98	1.30	0.90	1.41	0.83	1.52	0.75	1.65
30	0.05	1.35	1.49	1.28	1.57	1.21	1.65	1.14	1.74	1.07	1.83
	0.01	1.13	1.26	1.07	1.34	1.01	1.42	0.94	1.51	0.88	1.61
40	0.05	1.44	1.54	1.39	1.60	1.34	1.66	1.39	1.72	1.23	1.79
	0.01	1.25	1.34	1.20	1.40	1.15	1.46	1.10	1.52	1.05	1.58
50	0.05	1.50	1.59	1.46	1.63	1.42	1.67	1.38	1.72	1.34	1.77
	0.01	1.32	1.40	1.28	1.45	1.24	1.49	1.20	1.54	1.16	1.59
60	0.05	1.55	1.62	1.51	1.65	1.48	1.69	1.44	1.73	1.41	1.77
	0.01	1.38	1.45	1.35	1.48	1.32	1.52	1.28	1.56	1.25	1.60
80	0.05	1.61	1.66	1.59	1.69	1.56	1.72	1.53	1.74	1.51	1.77
	0.01	1.47	1.52	1.44	1.54	1.42	1.57	1.39	1.60	1.36	1.62
100	0.05	1.65	1.69	1.63	1.72	1.61	1.74	1.59	1.76	1.57	1.78
	0.01	1.52	1.56	1.50	1.58	1.48	1.60	1.46	1.63	1.44	1.65

附表 2. t-Table *t* 分配臨界值表

$$P(t > t_\alpha) = \alpha$$



d.f.	$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{.010}$	$t_{.005}$	*d.f.*
1	3.078	6.314	12.706	31.821	63.656	1
2	1.886	2.920	4.303	6.965	9.925	2
3	1.638	2.353	3.182	4.541	5.841	3
4	1.533	2.132	2.776	3.747	4.604	4
5	1.476	2.015	2.571	3.365	4.032	5
6	1.440	1.943	2.447	3.143	3.707	6
7	1.415	1.895	2.365	2.998	3.499	7
8	1.397	1.860	2.306	2.896	3.355	8
9	1.383	1.833	2.262	2.821	3.250	9
10	1.372	1.812	2.228	2.764	3.169	10
11	1.363	1.796	2.201	2.718	3.106	11
12	1.356	1.782	2.179	2.681	3.055	12
13	1.350	1.771	2.160	2.650	3.012	13
14	1.345	1.761	2.145	2.624	2.977	14
15	1.341	1.753	2.131	2.602	2.947	15
16	1.337	1.746	2.120	2.583	2.921	16
17	1.333	1.740	2.110	2.567	2.898	17
18	1.330	1.734	2.101	2.552	2.878	18
19	1.328	1.729	2.093	2.539	2.861	19
20	1.325	1.725	2.086	2.528	2.845	20
21	1.323	1.721	2.080	2.518	2.831	21
22	1.321	1.717	2.074	2.508	2.819	22
23	1.319	1.714	2.069	2.500	2.807	23
24	1.318	1.711	2.064	2.492	2.797	24
25	1.316	1.708	2.060	2.485	2.787	25
26	1.315	1.706	2.056	2.479	2.779	26
27	1.314	1.703	2.052	2.473	2.771	27
28	1.313	1.701	2.048	2.467	2.763	28
29	1.311	1.699	2.045	2.462	2.756	29
30	1.310	1.697	2.042	2.457	2.750	30
31	1.310	1.696	2.040	2.453	2.774	31
32	1.309	1.694	2.037	2.449	2.739	32
33	1.308	1.692	2.035	2.445	2.733	33
34	1.307	1.691	2.032	2.441	2.728	34
35	1.306	1.690	2.030	2.438	2.724	35
36	1.306	1.688	2.028	2.435	2.720	36
37	1.305	1.687	2.026	2.431	2.715	37
38	1.304	1.686	2.024	2.429	2.712	38
39	1.304	1.685	2.023	2.426	2.708	39
40	1.303	1.684	2.021	2.423	2.705	40