1. (15%) Find Laplace Transforms of the following functions u of time t:
   (a) (5%) $u(t)=e^{st}1(t)$ with s being a constant;
   (b) (5%) $u(t)=\cos(\omega t-\tau)1(t-\tau)$, $\omega,\tau>0$; and
   (c) (5%) the function u that is governed by $1(t)=u(t)+\int_0^t u(\tau)\,d\tau$,

   where 1 denotes the unit-step function of time.

2. (15%) Consider a dynamical system G, the transfer function of which is

$$\hat{G}(s)=\frac{1}{s^2+1}.$$

   (a) (5%) What is the impulse response of the system G?
   (b) (5%) What is the step response of the system G?
   (c) (5%) Can we apply the Final Value Theorem to obtain the DC gain of the system G? Why or why not?

3. (40%) Consider the tracking problem of the unity negative feedback system, as shown in Figure 1, where the plant is $G(s)=\dfrac{2}{s^2+s-2}$ with a proportional-integral (PI) feedback controller $H(s)=K_p+\dfrac{K_I}{s}$.
   (a) (15%) Determine the conditions on the P-gain K_p and I-gain K_I such that the closed-loop system is bounded-input-bounded-output (BIBO) stable.
   (b) (25%) Let $K_p=3$ and $K_I=1$. For each of the following reference commands, please determine the steady-state tracking error, i.e., $\lim_{t\to\infty} e(t)$. Note that the steady-state error may be constant in some cases and non-constant (time function) in some other cases.
      i. (7%) The unit step input, i.e., $r(t)=1$, $t\ge 0$.
      ii. (8%) The unit ramp input, i.e., $r(t)=t$, $t\ge 0$.
      iii. (10%) The sinusoidal input, i.e., $r(t)=\sin t$, $t\ge 0$.

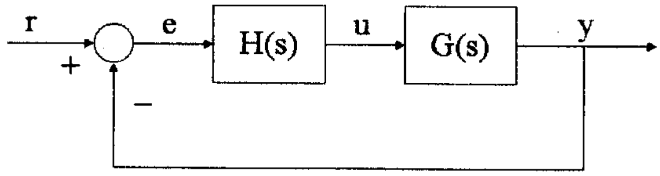

Figure 1

4. (30%) Given a feedback control system as shown in Figure 2, where *r* is the reference, *y* is the output and *e* is the error. G(s) is the plant. K is the controller which is a constant value.

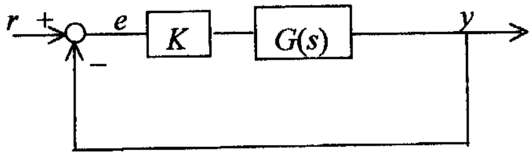

Figure 2

The transfer function *G(s)* is given as:

$$G(s)=\frac{50}{s(s+1)(s+10)}$$

(a) (10%) Please plot the root locus and find the breakaway points, asymptotes.

(b) (10%) Please plot the ***detailed*** Bode diagram (the magnitude vs. frequency and phase vs. frequency plots). Find the ***approximate*** value of the crossover frequency ω_c.

(c) (10%) Find the gain margin and determine the phase margin using the ***approximate*** ω_c obtained in (b). Please also determine the range of *K* to make the closed-loop system stable using the information obtained from the Bode diagram.