

國立中正大學 100 學年度 碩士班 招生考試 試題

系所別：電機工程學系 - 電磁晶片組  
計算機、機電工程組  
電力與電能處理組  
科目：線性代數與微分方程

第 2 節

第 1 頁，共 2 頁

微分方程部份 50 分

1. (10%) Solve the differential equation

$$(6xy - y^3)dx + (4y + 3x^2 - 3xy^2)dy = 0.$$

2. (10%) Solve the initial value problem

$$4y'' + 4y' + y = 0, \quad y(2) = 1, \quad y'(2) = 0.$$

3. (10%) Solve

$$\frac{dy}{dt} + y + \int_0^t y(u)du = 1$$

subject to  $y(0) = 0$ .

4. (10%) Find the general solution of the system

$$\mathbf{X}' = \begin{pmatrix} 3 & 1 \\ -1 & 1 \end{pmatrix} \mathbf{X} + \begin{pmatrix} -2 \\ 1 \end{pmatrix} e^{2t}$$

5. (10%) Solve the boundary-value problem

$$\begin{cases} y'' - y = 0, & 0 < x < 1 \\ y'(0) + 3y(0) = 0, & y'(1) + y(1) = 1 \end{cases}$$

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第 2 節

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線性代數部份 50 分

6. (10%) Find a matrix  $S$  such that  $S^2 = A$ , if  $A = \begin{bmatrix} 1 & 3 & 1 \\ 0 & 4 & 5 \\ 0 & 0 & 9 \end{bmatrix}$ .

7. (10%) Find the least squares solution of the linear system given by

$$\begin{aligned} x_1 & & -x_3 & = 6 \\ 2x_1 & +x_2 & -2x_3 & = 0 \\ x_1 & +x_2 & & = 9 \\ x_1 & +x_2 & -x_3 & = 3 \end{aligned}$$

8. (10%) What conditions must  $b_1$ ,  $b_2$ , and  $b_3$  satisfy in order for the following system of equations to be consistent?

$$\begin{aligned} x_1 & +2x_2 & +3x_3 & = b_1 \\ 2x_1 & +5x_2 & +3x_3 & = b_2 \\ x_1 & & +8x_3 & = b_3 \end{aligned}$$

9. (10%) Let  $\mathbf{u}$  and  $\mathbf{v}$  be nonzero vectors in 2- or 3-space, and let  $k = \|\mathbf{u}\|$  and  $l = \|\mathbf{v}\|$ . Show that the vector  $\mathbf{w} = l\mathbf{u} + k\mathbf{v}$  bisects the angle between  $\mathbf{u}$  and  $\mathbf{v}$  (i.e., the angles between  $\mathbf{u}$  and  $\mathbf{w}$  and between  $\mathbf{v}$  and  $\mathbf{w}$  are equal).

10. (10%) Find the coordinate vector of  $\mathbf{v}$  relative to the basis

$S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ , where

$$\mathbf{v} = (2, -1, 3), \quad \mathbf{v}_1 = (1, 0, 0), \quad \mathbf{v}_2 = (2, 2, 0), \quad \mathbf{v}_3 = (3, 3, 3)$$