招生學年度	105	招生類別	碩士班
系所班別	應用數學系 統計碩士班		
科目名稱	機率與統計		
注意事項	本考科禁止使用掌上型計算機；含機率論與統計學		

1. True or False ? In the following questions, decide whether the following statement is true or false. Justify/Explain your answer. Give an example or explain briefly if the answer is "True", otherwise, give a counterexample.

   (a) (5 points) If X is a continuous random variable with the probability density function $f(x), -\infty < x < \infty$, then $P(X = b) = f(b), \forall\, b$.

   (b) (5 points) If A and B are two disjoint events and $P(B) > 0$, then $P(A|B)$ must be 0.

   (c) (5 points) If the random variable X has a cumulative distribution function F, then $F(b + \frac{1}{n}) \to F(b)$ as $n \to \infty$ for any real value b.

   (d) (7 points) If $X_1, X_2, ..., X_n \sim_{\text{iid}} B(1,p)$, then $\overline{X} = \frac{\sum_{i=1}^{n} X_i}{n}$ is an unbiased estimator for p.

   (e) (8 points) If $X_1, X_2, ..., X_n \sim_{\text{iid}} N(0, \sigma^2)$, then $X_1^2 + X_2^2 + ... + X_n^2$ is a sufficient estimator for σ^2.

2. $X_1, X_2, ..., X_n \sim_{\text{iid}} N(\mu, \sigma^2)$ where μ is unknown and σ^2 is known.

   (a) (10 points) Please find the MLE (Maximum likelihood estimator) for μ and verify your answer.

   (b) (10 points) Please find the UMVUE (Uniformly minimum-variance unbiased estimator) for μ and verify your answer.

3. (15 points) $X_1, X_2, ..., X_n \sim_{\text{iid}} B(1, p_0)$. Please find the UMP (uniformly most powerful) test of size α for testing $H_0 : p = p_0$ v.s. $H_1 : p = p_1$, where $0 < p_0 < p_1 < 1$.

4. (10 points) Suppose X follows the uniform random variable on (-1,1). Please find the probability density function of X^2.

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5. Assume that Z is a standard normal random variable, and $g(x)$ is a differentiable and bounded function on $-\infty < x < \infty$. Please show

   (a) (10 points) $E(g'(Z)) = E(Zg(Z))$.

   (b) (5 points) $E(Z^{n+1}) = nE(Z^{n-1})$.

6. (10 points) Please interpret the "Law of Large Numbers".