國立中山大學100學年度碩士班招生考試試題

科目：工程數學【海下海物所碩士班選考】

1. (a) Find the Fourier expansion of the function whose definition in one period is, (10 %)

$$f(t)=4-t^2 \qquad -2\le t\le 2$$

(b) By the above results and simple sketch one period of $f(t)$, determine is this function odd or even? (10 %)

2. If the Laplace transform of $y(t)$ is (10 %)

$$\mathcal{L}\{y\}=\frac{s+1}{s^2+s-6}$$

What is $y(t)$?

3. Heat is generated at a constant rate r within a rod of finite length, the heat equation with boundary conditions are as follows, solve $u(x, t)$ (20 %)

$$k\frac{\partial^2 u}{\partial^2 x}+r=\frac{\partial u}{\partial t}$$

$$u(0,t)=0, \quad u(1,t)=u_0 \quad t>0$$

$$u(x,0)=f(x), \quad 0<x<1$$

4. S 為球面 $x^2+y^2+z^2=4$

$\vec{F}(x,y,z)=x\vec{i}+y\vec{j}+z\vec{k}$ ，

求 $\iint_S \vec{F}\bullet d\vec{S}$ 。（20%）

5. 求矩陣

$$\begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}$$

的特徵值(eigen value)及特徵向量(eigen vector)。（15%）

6. 求解常微分方程式

$\frac{d^2y}{dt^2}+2\frac{dy}{dt}+5y=-\sin t$ 。（15%）