國立中山大學100學年度碩士班招生考試試題

科目：微積分【海工系碩士班丙組選考】

Part 1: Differentiation and limits (50%)

1. (20%) Find the limit for each of the following questions **(5% each)**:

(a) $\lim_{x\to 2+}\frac{\ln(2x-3)}{x^2-4}$; (b) $\lim_{x\to\infty}\frac{2x-1}{\sqrt{3x^2+x+1}}$;

(c) $\lim_{x\to\infty}x^{1/x}$; (d) $\lim_{x\to 0+}\frac{\sin x}{x}$;

2. (10%) Find $\frac{du}{dt}$, given $u=x^2+2xy+y^2$ where $x=t\cos t$ and $y=t\sin t$.

3. (10%) Assume $u=f(x)$ and $v=f(x)$, and are differentiable at $x=0$, together with boundary conditions $u(0)=4$, $u'(0)=-3$, $v(0)=-2$ and $v'(0)=1$. Find:

(a) A complete mathematical expression for u and v; **(5%)**

(b) The numerical value of $\frac{d}{dx}\left(\frac{v}{u}\right)$. **(5%)**

[Note: The primes (') denote differentiation with respect to x.]

4. (10%) Sand is falling into a conical pile at the constant rate of 1.25×10^{-3} m^3 per second. If the height of the pile is always twice the radius of the base, determine the rate of increase in height when the pile is 0.3 m high?

[Note: The volume of the cone (pile) at any time can be expressed as

$V=\frac{1}{3}\pi R^2 h$, where R is the radius and h is vertical height.]

Part 2: Integration (50%)

5. (10%) Evaluate $\int\left(x^2\sqrt{x^3+4}\right)dx$

6. (10%) Evaluate $\int_0^1\left(x\tan^{-1}x\right)dx$

7. (10%) Evaluate $\int\left(x^{1/3}+x^{1/2}\right)^{-1}dx$

8. (10%) Evaluate $\frac{d}{dx}\int_{2x}^{x^2}u(1+u^2)^3\,du$

9. (10%) Find the area of the region bounded by the graph of

$y(x)=\frac{2x}{\sqrt{x^2+9}}$, with $y=0$, $x=0$ and $x=4$.