國立中山大學100學年度碩士班招生考試試題

科目：工程數學【海工系碩士班甲組】

1. 【Ordinary differential equations】20%

(a) Solve the IVP $x^2y'' + 3xy' + y = 0,\ y(1) = 4,\ y'(1) = -2$ (10%)

(b) Solve $y' + 4x^2y = (4x^2 - x)e^{-x^2/2}$ (10%)

2. 【Laplace transform】10%

Solve the IVP $y'' + 2y' + 2y = e^{-t} + 5\delta(t-2),\ y(0) = 0,\ y'(0) = 1$, where δ is the unit impulse function.

3. 【Linear algebra】10%

Given $\mathbf{A} = \begin{bmatrix} 5 & 4 \\ 4 & 11 \end{bmatrix}$ in an elastic deformation $\mathbf{y} = \mathbf{A}\mathbf{x}$, find the principal directions and corresponding factors of extension or contraction.

4. 【Vector calculus】10%

Calculate the work integral $\int_C \mathbf{F}(\mathbf{r}) \bullet d\mathbf{r}$, if $\mathbf{F} = [e^x \quad e^y]$ is a force and the work is done in the displacement clockwise along the circle with center (0, 0) from (1, 0) to (0, −1).

5. 【Partial differential equation】10%

Partial differential equations (PDE) may be classified as elliptic, parabolic or hyperbolic type.

(a) Give the general expression of a second-order linear PDE for variable $u(x,y)$ or $u(x,t)$. (3%)

(b) Give the criterion and solution (root) for each type of the PDE. (3%)

(c) Give one example of the mathematical equation and the common name for this equation in each type of the PDE mentioned above. (4%)

6. 【Method of separation of variables】15%

Apply the method of separation of variables to find all possible solutions for the PDE $u_{xx}^2(x,y) = u_y(x,y)$, where subscripts ($_{xx}$) and ($_y$) denote partial differentiation.

[Note: We may explicitly include the constant coefficients in the proposed solutions, because boundary conditions are not specified.]

7. 【Complex variables and Cauchy-Riemann equation】10%

(a) Given function $w = f(z) = u(x,y) + iv(x,y)$ in a complex domain, and the Cauchy-Riemann equations can be defined as $u_x = v_y$ and $u_y = -v_x$, where subscripts ($_x$) and ($_y$) denote partial differentiation. Is $f(z) = z^3$ analytic? (5%)

(b) Let $w = f(z) = z^2 + 3z$. Find u and v, and calculate the value of f at $z = 1 + 3i$ (5%)

國立中山大學100學年度碩士班招生考試試題

科目：工程數學【海工系碩士班甲組】

8. 【Fourier analysis】15%

Given a Fourier series for a periodic function by:

$$f(t)=\begin{cases}0, & -\pi<t<0,\\ t, & 0<t<\pi,\end{cases}$$

(a) Sketch the given function graphically and determine its period. (5%)

(b) Express the approximate solution in expanded form to the term including $n=4$. (10%)