

國立中山大學100學年度碩士班招生考試試題

科目：微積分【公事所碩士班甲組】

答案請按題號順序寫在答案卷上，違者扣分。

Are the following statements (1~10) true or false? Give an explanation for your answer or a counterexample if the answer is false.

1. There is a function which is continuous on $[0,4]$ but not differentiable at $x=3$. 5%
2. If a function is continuous, then it is differentiable. 5%
3. If $f(a) \neq g(b)$, then $f'(a) \neq g'(b)$. 5%
4. A critical point of f must be a local maximum or minimum of f . 5%
5. The derivative of a polynomial is always a polynomial. 5%
6. Since $f(x)=1/x$ is continuous for all $x>0$, and the interval $(0,1)$ is bounded, f has a maximum on the interval $(0,1)$. 5%
7. Suppose f'' and g'' exist and f and g are concave up for all x . Then $f(g(x))$ is concave up for all x . 5%
8. If $f'(x)=g'(x)$, then $f(x)=g(x)$. 5%
9. If $f''(x)>0$, then $f'(x)$ is increasing. 5%
10. If $g'(a) \neq 0$, then $\lim_{x \rightarrow a} f(x)/g(x)=f'(x)/g'(x)$. 5%

Calculate the following (11~18) 5% each.

$$11. \int_0^1 \int_y^1 e^{x^2} dx dy$$

$$12. \int (x^2 e^{x^3+1}) dx$$

$$13. \frac{d}{dx} \int_0^{x^2} \sin t^2 dt$$

$$14. \int (3e^x + 5 \sin x) dx$$

$$15. \int \frac{2}{\sqrt{x}} dx$$

$$16. \int \frac{x+3}{x} dx$$

$$17. \int (x^4 e^{2x}) dx$$

$$18. \text{Approximate } \int_0^1 \sqrt{6-x^2} dx \text{ (hint: using Taylor series)}$$

19. You run a small furniture business. You sign a deal with a customer to deliver up to 400 tables, the exact number to be determined by the customer later. The price will be \$900 per table up to 300 tables, and above 300, the price will be reduced by \$2.5 per table (on the whole order) for every additional chair over 300 ordered. What are the largest and smallest revenues your company can make under this deal? 10%