

## 國立中山大學100學年度碩士班招生考試試題

## 科目：微積分丁【企管系甲班碩士班丁組選考】

答案請按題號順序寫在答案卷上，違者扣分。

Are the following statements (1~10) true or false? Give an explanation for your answer or a counterexample if the answer is false.

1. There is a function which is continuous on  $[0,4]$  but not differentiable at  $x=3$ . (5%)
  2. If a function is continuous, then it is differentiable. (5%)
  3. If  $f(a) \neq g(b)$ , then  $f'(a) \neq g'(b)$ . (5%)
  4. A critical point of  $f$  must be a local maximum or minimum of  $f$ . (5%)
  5. Since  $f(x)=1/x$  is continuous for all  $x>0$ , and the interval  $(0,1)$  is bounded,  $f$  has a maximum on the interval  $(0,1)$ . (5%)
  6. If  $g'(a) \neq 0$ , then  $\lim_{x \rightarrow a} f(x)/g(x) = f'(x)/g'(x)$ . (5%)
  7. The derivative of a polynomial is always a polynomial. (5%)
  8. Suppose  $f''$  and  $g''$  exist and  $f$  and  $g$  are concave up for all  $x$ . Then  $f(g(x))$  is concave up for all  $x$ . (5%)
  9. If  $f'(x) \leq g'(x)$ , then  $f(x) \leq g(x)$ . (5%)
  10. If  $f''(x) > 0$ , then  $f'(x)$  is increasing. (5%)
- Calculate the following (11~18) (5% each.)

$$11. \int \frac{5}{\sqrt{x}} dx$$

$$12. \int \frac{x+1}{x} dx$$

$$13. \int (x^4 e^{3x}) dx$$

$$14. \int (3e^x + 2\sin x) dx$$

$$15. \int (x^2 e^{x^3+1}) dx$$

$$16. \int_0^1 \int_y^1 e^{x^2} dx dy$$

$$17. \frac{d}{dx} \int_2^{x^2} \sin(t^2) dt$$

18. A function  $f$  has  $f(3)=1$ ,  $f'(3)=7$  and  $f''(3)=-10$

Find the best estimate you can for  $f(3.1)$

19. You run a small furniture business. You sign a deal with a customer to deliver up to 400 tables, the exact number to be determined by the customer later. The price will be \$900 per table up to 300 tables, and above 300, the price will be reduced by \$2.5 per table (on the whole order) for every additional chair over 300 ordered. What are the largest and smallest revenues your company can make under this deal? (10%)