

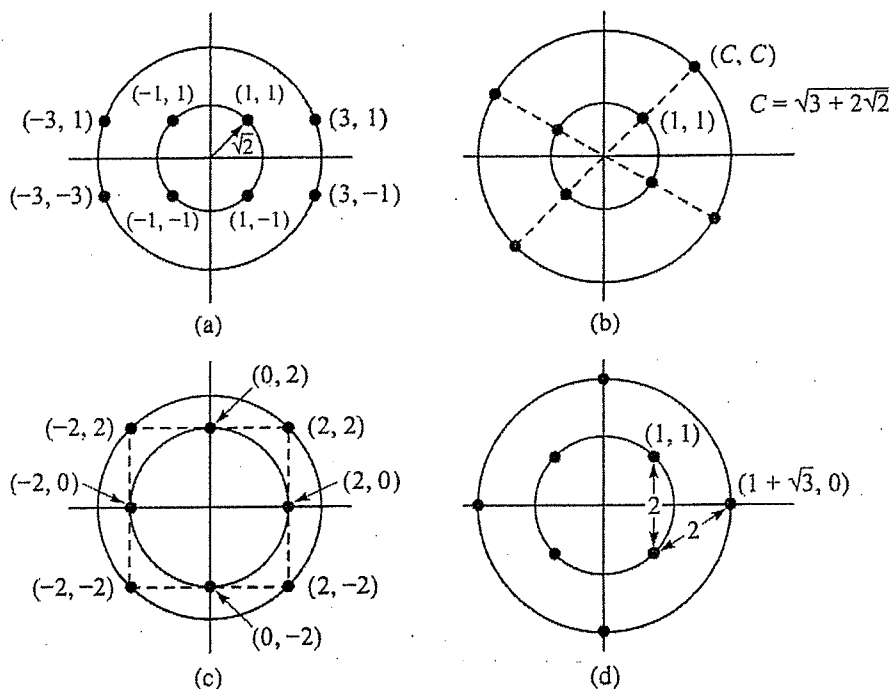
通訊理論 (Communications Theory)

1. [20] **Nyquist Pulse-Shaping Criterion:** Let the symbol duration $T = 1$. Which of the following signals are Nyquist pulses for zero-ISI? Please explain your answers.

- (a)[4] $\frac{\sin \pi t}{\pi t}$ (b)[4] $\frac{\sin \pi t}{\pi t} \exp\{-2t^2\}$ (c)[4] $\sin \pi t + \cos \pi t$ (d)[4] $\exp\left\{-\frac{t^2}{2}\right\}$ (e)[4] $\left(\frac{\sin \pi t}{\pi t}\right)^4$

2. [20] **Signal Constellation:**

- (a) [10] Please design the Gray coding for an 8-PSK modulation scheme.
 (b) [10] Which of the following 8-QAM modulation schemes has the best performance? Please explain your answer. (Assume that all signal points are equally probable.)



3. [20] **Optimum Receivers for AWGN Channels:** Please derive the error probability for M -ary biorthogonal signaling that adopts optimal detection. All signals are equiprobable and have equal energy. The noises are i.i.d. Gaussian random variables with zero-mean and variance $\frac{N_0}{2}$. (You do not have to show a closed form expression.)

4. [20] **Explanations:**

- (a) [2] What is an energy signal?
 (b) [2] What is a power signal?
 (c) [2] What kind of system is said to be causal?
 (d) [2] What kind of system is said to be stable?
 (e) [2] What is an Ergodic process?
 (f) [2] What is a cyclostationary process?
 (g) [2] Please describe the condition that two events are statistically independent.
 (h) [2] Please describe the condition for two random variables X and Y to be orthogonal.
 (i) [2] Please describe the conditions for a random process to be wide-sense stationary (WSS).
 (j) [2] Please describe the Offset QPSK modulation scheme.

5. [20] **Fourier Transform:** (Hint: You may use the attached properties of the Fourier transform.)
 (a) [10] Please show that the Fourier transform of a decaying exponential pulse is given by:

$$\exp(-at)u(t) \Leftrightarrow \frac{1}{a + j2\pi f}, a > 0, \text{ where } u(t) = \begin{cases} 1, & t > 0 \\ \frac{1}{2}, & t = 0 \\ 0, & t < 0 \end{cases}$$

- (b) [10] Please show that the Fourier transform of a double exponential pulse is given by:

$$\exp(-a|t|) \Leftrightarrow \frac{2a}{a^2 + (2\pi f)^2}, a > 0.$$

Properties of the Fourier Transform

Property	Mathematical Description
Linearity	$ag_1(t) + bg_2(t) \Leftrightarrow aG_1(f) + bG_2(f)$, where a and b are constants.
Time scaling	$g(at) \Leftrightarrow \frac{1}{ a } G\left(\frac{f}{a}\right)$, where a is a constant.
Duality	If $g(t) \Leftrightarrow G(f)$, then $G(t) \Leftrightarrow g(-f)$.
Time shifting	$g(t - t_0) \Leftrightarrow G(f) \exp(-j2\pi ft_0)$.
Frequency shifting	$\exp(j2\pi f_c t) g(t) \Leftrightarrow G(f - f_c)$.
Area under $g(t)$	$\int_{-\infty}^{\infty} g(t) dt = G(0)$.
Differentiation in the time domain	$\frac{d}{dt} g(t) \Leftrightarrow j2\pi f G(f)$.
Integration in the time domain	$\int_{-\infty}^{\infty} g(\tau) d\tau \Leftrightarrow \frac{1}{j2\pi f} G(f) + \frac{G(0)}{2} \delta(f)$.
Conjugate functions	If $g(t) \Leftrightarrow G(f)$, then $g^*(t) \Leftrightarrow G^*(-f)$.
Multiplication in the time domain	$g_1(t) g_2(t) \Leftrightarrow \int_{-\infty}^{\infty} G_1(\lambda) G_2(f - \lambda) d\lambda$.
Convolution in the time domain	$\int_{-\infty}^{\infty} g_1(\tau) g_2(t - \tau) d\tau \Leftrightarrow G_1(f) G_2(f)$.
Rayleigh's energy theorem	$\int_{-\infty}^{\infty} g(t) ^2 dt = \int_{-\infty}^{\infty} G(f) ^2 df$.