

## 國立中山大學100學年度碩士班招生考試試題

科目：機率【通訊所碩士班甲組】

1. (10%) A pair of fair dice is rolled six times. What is the probability that “nine” will not show at all?
2. (10%) Let  $X$  be a random variable with exponential distribution, derive and find the mean and variance of  $6X$ .
3. (10%) Let  $X$  be a normal random variable with mean 10 and variance 9. How can we design a random variable  $Y$  such that  $Y$  is normal distributed with mean 100 and standard deviation 2.
4. (10%) Let  $X$  and  $Y$  be independent uniform random variables with ranges  $[0, 2]$  and  $[0, 6]$  respectively. Derive and plot the probability density function of the random variable  $Z = X + Y$ .
5. (10%) Let  $X$  and  $Y$  be independent normal random variables with mean 0 and variance 6. Derive and plot the probability density functions of the random variables  $Z = \sqrt{X^2 + Y^2}$  and  $W = Y/X$ .
6. (10%) Given a real-valued random variable  $X$  with finite second moment. Identify all the true statements:
  - (a)  $E\{X^2\} \leq (E\{X\})^2$  ;
  - (b)  $E\{cX\} \neq cE\{X\}$ , where  $c$  is a constant value;
  - (c)  $E\{\log(1 + X)\} \leq \log(1 + E\{X\})$ .
7. (10%) Given a condition so that the fact  $E\{\frac{1}{X}\} = \frac{1}{E\{X\}}$  is true.
8. (10%) Let  $X$  and  $Y$  be independent normal random variables with zero mean and unit variance. Find the value of  $E\{X^2Y + XY^2\}$ , in which  $E\{\cdot\}$  takes the expectation with respect to  $X$  and  $Y$ .
9. (10%) Given a real-valued random variable  $X$  with finite second moment  $E\{X^2\}$ . Show the conditions on  $c$  so that the following statement is true:
 
$$E\{X^2\} \leq cE\{X^2\} \text{ if and only if } E\{X^2\} = 0.$$
10. (10%) Let  $Y$  be a binomial distribution with parameters  $n$  and  $p$ ; i.e., the probability distribution function of  $Y$  is given by  $P(Y = y) = \binom{n}{y} p^y (1-p)^{n-y}$ ,  $y = 0, 1, 2, \dots, n$ . Show the probability generating function of  $Y$ .