國立中山大學100學年度碩士班招生考試試題

科目：工程數學【資工系碩士班乙組】

1. (10%) Solve the following exact differential equation:

$$(y^2 + 1)dx + (2xy + 4)dy = 0$$

2. (15%) Laplace Transform

2.1 (10%) Solve the following equation by Laplace transform:

$$y(t) = t + \int_0^t y(t-\tau)\sin\tau\, d\tau$$

2.2 (5%) Find the inverse Laplace transform of

$$F(s) = \frac{s}{(s-2)(s^2+4s+5)}$$

3. (25%)

3.1 (10%) Given $f(x)=x$, $g(x)=x^2$, find the *Inner Product* and *norms* of $f(x)$ and $g(x)$ on the interval $0 \le x \le 1$.

3.2 (5%) Let $\mathbf{B}=\begin{bmatrix} 2 & 1 & -1 \\ 1 & -3 & 1 \\ 1 & 3 & -3 \end{bmatrix}$, find $\mathbf{B}^{-1}$.

3.3 (5%) Find the rank of matrix $\mathbf{B}$, *Rank*($\mathbf{B}$), where $\mathbf{B}=\begin{bmatrix} 3 & 1 & 4 & 0 \\ 1 & 0 & 1 & -2 \\ 2 & 1 & 3 & 2 \end{bmatrix}$.

3.4 (5%) An $n \times n$ matrix $\mathbf{B}$ is orthogonal if $\mathbf{B}^{-1} = \mathbf{B}^T$. Determine whether $\mathbf{B}=\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{-2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$ is orthogonal and find $\mathbf{B}^{-1}$.

4. (20%) Let $\mathbf{B} = \begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix}$ and $f(x)=x^5-3x^4+4x^3-2x^2+6x-2$.

4.1. (10%) Find $f(\mathbf{B})$.

4.2. (5%) Find the eigenvalues of $f(\mathbf{B})$.

4.3. (5%) What is the determinant of $f(\mathbf{B})$ (i.e., $|f(\mathbf{B})|$)?

5. (20%) Fourier Analysis

5.1 (10%) Find the Fourier coefficients of the periodic function

$$f(t) = \begin{cases} 0 & -\pi < t < 0 \\ \pi & 0 < t < \pi \end{cases}$$

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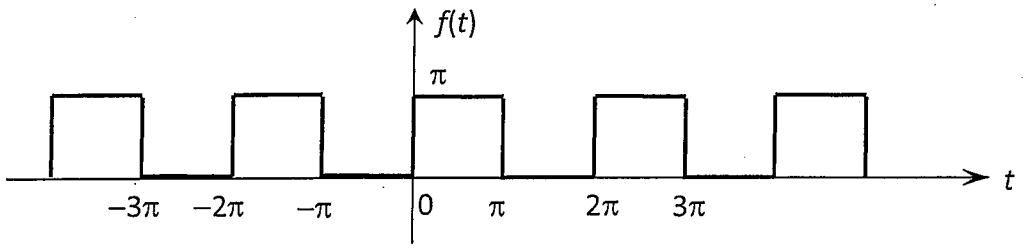

5.2 (5%) The Fourier transform $X(\omega)$ of $x(t)$ is defined as $X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$.

Find the Fourier transform of the following function

$$x(t) = \begin{cases} 0 & t < -1 \\ 1 & -1 < t < 1 \\ 0 & t > 1 \end{cases}$$

5.3 (5%) The inverse Fourier transform $x(t)$ of $X(\omega)$ is defined as

$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega$. Use the result of Problem 5.2, prove that

$$\int_0^{\infty} \frac{\sin\omega}{\omega} d\omega = \frac{\pi}{2}.$$

6 (10%) z-Transform

The z-transform of a discrete data sequence $x[n] = \{x_0, x_1, \ldots\}$ is defined as:

$$X(z) = \sum_{n=0}^{\infty} x_n z^{-n}$$

6.1 (3%) Derive the z-transform of sequence $x[n]$ where $x_n = b^n$ and b is a constant.

6.2 (7%) Let z-transform of a sequence $y[n]$ be denoted as $Z\{y[n]\}$. Show that $Z\{y[n-1]\} = z^{-1} \cdot Z\{y[n]\}$, and solve the following difference equation by using z-transform:

$$y[n] - \frac{1}{2}y[n-1] = \delta[n]$$

where $\delta[n]$ is the delta function.