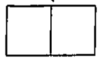
國立中山大學100學年度碩士班招生考試試題

科目：離散數學【資工系碩士班甲組】

There are 8 problems in this test. Write down detailed steps for the solution to each problem. Otherwise, no credits for that problem will be given.

1. (10%) Let $m > 1$ and $n > 1$ be two positive integers. Let $r(m, n)$ denotes the maximum number of rectangles defined by m horizontal lines and n vertical lines in a plane. Derive a formula for $r(m, n)$. Note that rectangles may overlap. For example, let $m = 2$ and $n = 3$ (), $r(2, 3) = 3$, not 2.

2. (10%) Let $x_1, x_2, \ldots, x_n$ be a sequence of n integers. A consecutive subsequence of $x_1, x_2, \ldots, x_n$ is a subsequence $x_i, x_{i+1}, \ldots, x_j$ for some $1 \leq i \leq j \leq n$. Show that for any k, $1 \leq k \leq n$, there is a consecutive subsequence whose sum is divisible by k.

3. (10%) Let the sequence of numbers $g_0, g_1, \ldots, g_n, \ldots$ be defined by $g_0 = 1$, $g_1 = 1$ and, for every $n > 1$, $g_n = g_{n-1} + 2g_{n-2} + (-1)^n$. Express g_n in terms of n.

4. (10%) A *planar* graph is a graph which can be embedded in a plane without crossing edges. Let $G = (V, E)$ be a simple graph with n vertices and m edges.

   (a) (5%) Show that if G is planar with $n > 2$, then $m \leq 3n - 6$.

   (b) (5%) Define G^c, the complement of G, to be the graph with the vertex set V. For every pair of vertices x and y, the edge xy is in G^c if and only if xy is not in G. Show that if G is planar with $n > 10$, then G^c is not planar.

5. (10%) Simplify the Boolean function $(f + g + h)(f + g + \bar{h})(f + \bar{g} + h)$, by using

   (a) (5%) the laws of Boolean algebra, and

   (b) (5%) the method of Karnaugh maps.

6. (10%) A *tree* is a connected graph without cycles.

   (a) (5%) Show that if G is a tree of more than 1 vertex, then G has at least 2 vertices of degree 1.

   (b) (5%) Can there be a stronger theorem:

   > If G is a tree of more than α vertices, then G has at least β vertices of degree 1.

   for some α and $\beta > 2$. Justify your answer.

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7. (20%) Let $S = \{1, 2, \ldots, n\}$. Define a relation $\sim$ on S, such that $x \sim y$ if and only if $x = 2^k y$ for some integer k.

   (a) (5%) Show that the relation $\sim$ is an equivalence relation.

   (b) (5%) Show the equivalence classes for $n = 20$ and $n = 25$, respectively.

   (c) (10%) Let $\lfloor x \rfloor$ be the largest integer less than or equal to x. Show that if $\left\lfloor \frac{n+1}{2} \right\rfloor + 1$ numbers are chosen from the set S, then there must be two numbers a and b such that a is divisible by b.

8. (20%) Let m and n be two positive integers, $m \leq n$. Define $\binom{n}{m} = \dfrac{n!}{m!(n-m)!}$.

   (a) (10%) Show that if n is prime, then n divides $\binom{n}{i}$ for every i, $1 \leq i < n$.

   (b) (10%) Show that if n is composite, then n does not divide $\binom{n}{i}$ for some i, $1 \leq i < n$.