國立中山大學100學年度碩士班招生考試試題

科目：工程數學【機電系碩士班】

I. (35%)

1. (25%) Solve the given differential equation by means of a power series about the given point x_0.

(a) Find the recurrence relation; (5%)

(b) Find the first four terms in each of two linearly independent solutions (unless the series terminates sooner). (10%)

(c) Find the general term in each solution. (10%)

$$y'' + y = 0, \quad x_0 = 0$$

2. (10%) $y''' - 3y'' + 2y' = t + e^t, \quad y(0) = 1, \ y'(0) = -\frac{1}{4}, \ y''(0) = -\frac{3}{2}$

(a) Find the solution of the given initial value problem. (5%)

(b) Then draw a graph of the solution. (5%)

II. (30%)

1. (10%) In a certain state, 25% of all cars emit excessive amount of pollutants. If the probability is 0.99 that a car emitting excessive amount of pollutants will fail the state's vehicular emission test, and the probability is 0.17 that a car not emitting excessive amount of pollutants will nevertheless fail the test, what is the probability that a car which fails the test actually emits excessive amount of pollutants?

2. (10%) One baseball team won only 41% of their games in the 2010 season. Assuming that this team has a 41% chance of winning each game next season, what is the probability that this team can win exactly 15 games in the first 20 games of the 2011 season?

3. (10%) Please find $(AB)^{-1}$ for

$$A = \begin{pmatrix} -1 & 2 \\ 2 & 1 \end{pmatrix} \qquad B^{-1} = \frac{1}{4}\begin{pmatrix} -4 & 0 \\ 4 & 1 \end{pmatrix}$$

III. (35%)

1. (20%) Calculate the line integral $\oint_C \mathbf{F} \bullet \mathbf{r}' ds$ clockwise as seen by a person standing at the origin, where $\mathbf{F} = [-3y, 3x, z]$, $\mathbf{r}'$: the tangent vector, $\bullet$: inner product operation, ds: line increment, C: the circle of $x^2 + y^2 = 4$, $z = 1$.

2. (15%)

(a) Find the Fourier series of $f(x)$, where $f(x) = -k$ if $-1 < x < 0$ and $f(x) = k$ if $0 < x < 1$ with period p=2. (10%)

(b) Find the sum of $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$ (5%)