國立中山大學100學年度碩士班招生考試試題

科目：資料結構【電機系碩士班丙組選考】

1. (a) [5 points] Given an input size n, where n is a positive integer, we assume that a program requires the running time $\Theta(f(n))$. State the *formal* definition of $\Theta(f(n))$.

   (b) [10 points] Given an input size n, where n is a positive integer, we assume that the program requires the running time $T(n) = \Theta(f(n))$, where

   $$T(n) = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}.$$

   Derive $f(n)$ in the *simplest* formula.

2. (a) [5 points] The sequence F_n of Fibonacci numbers is defined as follows.

   $$F_n = \begin{cases} 1 & \text{if } n = 0 \text{ or } n = 1, \\ F_{n-1} + F_{n-2} & \text{if } n \geq 2. \end{cases}$$

   The FIBONACCI NUMBER PROBLEM is defined as "Given an integer $n \geq 0$, output the n-th Fibonacci number F_n." The following function **`Fib(int n)`** can solve the FIBONACCI NUMBER PROBLEM.

```
int Fib(int n) {
  if ((n==0) || (n==1))
    return 1;
  else
    return Fib(n-1)+Fib(n-2);
}
```

   Prove that the running time $T(n)$ of **`Fib(n)`** is larger than $\left(1+\frac{1}{\sqrt{5}}\right)\left(\frac{1+\sqrt{5}}{2}\right)^{n-2}$.

   (b) [5 points] Now, please tell us whether the FIBONACCI NUMBER PROBLEM is NP-complete? Explain your reasons. (Note that you will get 0 points if you do not present any reasons.)

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3. (a) [4 points] Given an unsorted integer array of size n, does the binary search algorithm perform better than the sequential search algorithm? Use the big-O notation to justify your answer.

   (b) [10 points] Given an integer array of size n, show that any comparison-based sorting algorithm requires a running time of $\Omega(n \log n)$ in the worst case.

   (c) [10 points] Given an unsorted integer array `A[n]` of size n, the following shows the quick sort algorithm, where we assume that the function `medium(array A)` can return the *medium* from the integer array `A[n]` in $\Theta(n)$ time. Note that given a set of n elements, the median is defined as the $\lceil n/2 \rceil$ largest element in that set.

   Derive the worst case running time of `quick_sort(array A)` in terms of Θ notation. (Note that you will get 0 points if you just give the answer directly.)

```
quick_sort(array A) {
  int x;
  if ( size[A] == 0 )
    return;
  // That is, if array A contains no element, do nothing.
  x = medium(A);
  S = { y | y∈A and y≤x };
  L = { z | z∈A and z>x };
  quick_sort(S);
  print x;
  quick_sort(L);
}
```

   (d) [5 points] Now, suppose that we want to sort an integer array `B[1024]`. Derive the worst case running time of `quick_sort(B)` in terms of Θ notation. (Note that you will get 0 points if you just give the answer directly.)

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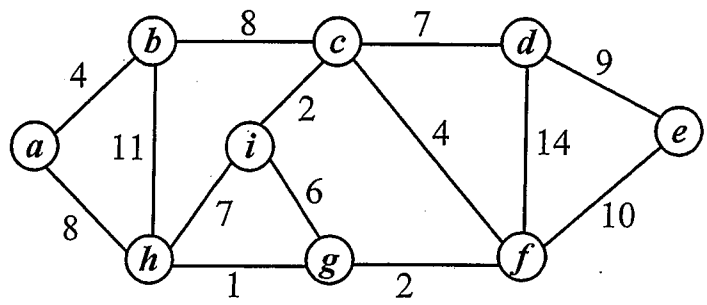
4. Consider the following weighted graph.



(a) [5 points] Star from the vertex ***a*** and use Prim's algorithm to find the minimum cost spanning tree. Show the actions step by step.

(b) [5 points] Use Kruskal's algorithm to find the minimum cost spanning tree. Show the actions step by step.

5. [12 points] Complete the following notation translations.

Infix	Prefix	Postfix
`a*(b+c*d)/e-f`		
		`ab/cd+e-*fg-+`
	`*/a-*bc+de-fg`	

6. Insert a sequence of keys `{30, 43, 14, 20, 47, 25, 55, 40, 51, 6, 35}`, in that order, into a data structure which has no keys initially.
   (a) [5 points] Construct a binary search tree for that sequence.
   (b) [5 points] Construct an AVL tree for that sequence.
   (c) [5 points] Construct a heap tree for that sequence. Note that, in the question 6(b), we require that the root must have the maximum key value.

7. Given a hash table of size 11 (assuming that the hash table starts with the index 0), use the function $h(key) = (2 \times key + 5) \bmod 11$ to hash the following keys: `14, 43, 17, 81, 23, 91, 19, 20, 65,` and `8`. Draw the results with two different ways of handling collisions.
   (a) [4 points] Collisions are handled by chaining.
   (b) [5 points] Collisions are handled by linear probing.