國立中山大學100學年度碩士班招生考試試題

科目：離散數學【電機系碩士班丙組選考】

1. Define or explain the following terms: [20%]
   (a) The well-ordering principle
   (b) The fundamental theorem of arithmetic
   (c) The four color theorem
   (d) A relation on a set

2. For a group with n persons prove that at least two of them know the same number of persons in the group. Assume that if A knows B then B also knows A.
   (Hint: Use a node to represent a person. If two persons know each other, the corresponding nodes are connected with an edge. Consider the situation of the degree of each node.) [10%]

3. Let A={1,2,3,4,5,6,7}. For each of the following values of r, determine an equivalence relation $\mathcal{R}$ on A with $|\mathcal{R}| = r$, or explain why no such relation exists. [15%]
   (a) $r = 20$ (b) $r = 11$ (c) $r = 31$.

4. Prove that $K_{3,3}$ is nonplanar. [10%]

5. For $A = \{a, b, c, d, e\}$, the Hasse diagram for the poset $(A, \mathcal{R})$ is shown in the Fig. 1.
   (a) Determine the relation matrix for $\mathcal{R}$. [5%]
   (b) Topologically sort the poset $(A, \mathcal{R})$. [5%]

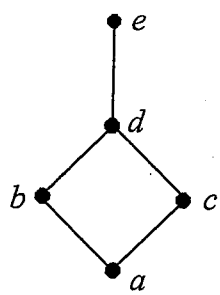

Fig. 1

6. Solve the following recurrence relation. [10%]
   $a_n - 6a_{n-1} + 9a_{n-2} = 0,\ n \geq 2,\ a_0 = 5,\ a_1 = 12$

7. For the graph in Fig. 2,
   (a) Find the chromatic polynomial by decomposition theorem. **(Note: Do not answer only by inspection.)** [10%]
   (b) What is its chromatic number and why? [5%]
   (c) Draw the dual graph of it. [10%]

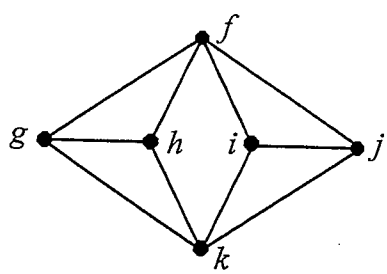

Fig. 2