國立中山大學100學年度碩士班招生考試試題

科目：數值分析【應數系碩士班乙組】

Twenty points for each problem. Please write down all the detail of your computation and answers.

1. (1)Why a subtraction of two close floating point numbers is unstable? Please give an example to illustrate it.

   (2)For very small number $\varepsilon \approx 0$, how to compute $\sqrt[3]{x+\varepsilon} - \sqrt[3]{x}$ and $\sin(x+\varepsilon) - \sin x$ to avoid unstable substraction.

2. Write a program to evaluate the polynomial

$$f(x) = \sum_{i=0}^{n} a_i x^i$$

   and its derivative at $x = c$ using least arithmetic operation. Compute the number of arithmetic operation needed in your program.

3. (1)Under what conditions will a fixed point iteration

$$x_n = g(x_{n-1}), \; n = 1, 2, \cdots$$

   converge?

   (2)Let $a > 0$. Show that the sequence

$$x_n = \frac{x_{n-1}}{2} + \frac{a}{2x_{n-1}}, \; n = 1, 2, \cdots$$

   converges to $\sqrt{a}$ for all $x_0 > 0$. What happens if $x_0 < 0$?

4. Use polynomial interpolation to prove the three-point midpoint formula for second derivative

$$f''(c) \approx \frac{1}{h^2}[f(c+h) - 2f(c) + f(c-h)],$$

   and use Taylor's Theorem to compute its error formula. Is this a stable method?

5. Write the following matrix A as the PLU decomposition $A = PLU$, where P is a permutation matrix, L lower triangular matrix, and U upper triangular matrix.

$$\begin{pmatrix} 1 & -2 & 3 & 0 \\ 1 & -2 & 3 & 1 \\ 1 & 3 & 2 & -2 \\ 2 & 1 & 3 & -1 \end{pmatrix}$$