國立中山大學100學年度碩士班招生考試試題

科目：微積分【應數系碩士班乙組】

計算題：共7題，子題分數平均分配。答題時，每題都必須寫下題號與詳細步驟。

[1]. (16%) Evaluate the following limits.

(a) $\lim\limits_{x\to\infty} \left(\dfrac{x}{x+3}\right)^x$

(b) $\lim\limits_{(x,y)\to(3,6)} \dfrac{x+y-9}{\sqrt{x+y}-3}$

[2]. (12%) Find the first derivative $F'(x)$ and the second derivative $F''(x)$ of

$$F(x) = \int_x^{x^2} \frac{\tan^{-1}\theta}{\theta}\, d\theta, \text{ where } \tan^{-1}\theta \text{ denotes the inverse function of } \tan\theta.$$

[3]. (16%) Consider the finite region in the first quadrant bounded by the curves $y=x^2$ and $y=4x$. Formulate the following quantities by integral. (**Do not need to evaluate.**)
   (a) the area of the region.
   (b) the volume of the solid obtained by rotating the region around the x-axis.
   (c) the perimeter of the region.
   (d) the volume of the solid obtained by rotating the region around the line $x=4$.

[4]. (15%) Evaluate the integral.

$$\int_0^\infty \frac{1}{(x+1)(x^2+1)}dx$$

[5]. (13%) Determine the interval of convergence of the power series.

$$\sum_{n=0}^\infty \frac{(-3)^n (x+1)^n}{\sqrt{n+1}}$$

[6]. (12%) Find the local maximal value and saddle point of $f(x,y) = 6x^2 - 2x^3 + 3y^2 + 6xy + 11$.

[7]. (16%) Consider the Cartesian integral

$$\int_{-1}^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \frac{4}{1+x^2+y^2}\, dxdy.$$

(a) Change the Cartesian integral into an equivalent polar integral.
(b) Evaluate the polar integral.

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