

## 國立中山大學100學年度碩士班招生考試試題

科目：線性代數【應數系碩士班乙組、丙組】

1. (60 points) Let  $A = \begin{bmatrix} 7 & -6 \\ 6 & -5 \end{bmatrix}$ .

- (a) (10 points) Is  $A$  diagonalizable?(Give your reasons).  
 (b) (10 points) Find the characteristic polynomial of  $A^{33}$ .(Give your reasons)  
 (c) (10 points) Find the eigenvalues of  $A^{66}$ .(Give your reasons)  
 (d) (10 points) Find the minimal polynomial of  $A$ .(Give your reasons)  
 (e) (10 points) Find the minimal polynomial of  $A^{99}$ .(Give your reasons)  
 (f) (10 points) Find the Jordan form of  $A$ .(Give your reasons)

2. (10 points) Prove or disprove that  $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$  and  $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$  is similar.

3. (30 points) Let  $U, V, W$  be vector spaces, and let  $S : U \rightarrow V, T : V \rightarrow W$  be linear maps. Let  $C = \text{Im}S \cap \text{Ker}T$  (i.e.  $C$  is the intersection of Image of  $S$  and Kernel of  $T$ ).

Prove or disprove the followings:

- (a) (10 points)  $C$  is a vector subspace of  $V$ .  
 (b) (10 points)  $\dim C = \dim \text{Ker}TS - \dim \text{Ker}S$ .  
 (c) (10 points)  $\dim \text{Im}TS \leq \dim \text{Im}S$ .