

## 國立中山大學100學年度碩士班招生考試試題

科目：基礎數學【應數系碩士班甲組】

共七題。答題時，每題都必須寫下題號與詳細步驟。

請依題號順序作答，不會作答題目請寫下題號並留空白。

1. (10%) Evaluate  $\lim_{n \rightarrow \infty} n(\sqrt[n]{2} - 1)$ .

2. (10%) Evaluate  $\int_1^{64} \frac{dx}{\sqrt{x} + \sqrt[3]{x}}$ .

3. (15%) Show that  $2 \sin x + \cos x = x$  has three solutions in  $\mathbb{R}$ .

4. (15%) Calculate the area of the region  $\Omega$  enclosed by the curves

$$4x^2 + 4xy + y^2 - 3x + 3y = 18 \quad \text{and} \quad 4x^2 + 4xy + y^2 + 3x - 3y = 18.$$

5. (15%) Let  $(\mathbb{R}^3, \langle \cdot, \cdot \rangle)$  be an inner product space with  $\langle u, v \rangle = u_1v_1 + u_2v_2 + u_3v_3$ ,  $u = (u_1, u_2, u_3)$  and  $v = (v_1, v_2, v_3)$ . Let  $V$  be the subspace of  $\mathbb{R}^3$  spanned by  $(2, 1, 1)$  and  $(1, -1, 2)$ . Find an orthonormal basis for  $V$  that contains the vector  $(2/\sqrt{6}, 1/\sqrt{6}, 1/\sqrt{6})$ .

6. (15%) Let  $A, B$  and  $A + B$  be invertible  $m \times m$  matrices. Denote the inverse matrices of  $A, B$  and  $A + B$  by  $A^{-1}, B^{-1}$  and  $(A + B)^{-1}$ , respectively. Show that the inverse matrix of  $A^{-1} + B^{-1}$  is  $A(A + B)^{-1}B$ .

7. (20%) Let the sequence  $\{a_k\}_{k \geq 0}$  be given by  $a_0 = 1$  and  $a_{k+1} = a_k + \frac{1}{a_k}$  for  $k \geq 0$ . Show that the sequence  $\{a_k - \sqrt{2k}\}_{k \geq 0}$  converges.