

國立中央大學100學年度碩士班考試入學試題卷

所別：工業管理研究所碩士班 乙組(一般生) 科目：作業研究 共 2 頁 第 1 頁

本科考試禁用計算器

\*請在試卷答案卷(卡)內作答

1. (20分) John has a linear programming (LP) model as follows.

$$\begin{aligned} & \text{Minimize } cx \\ & \text{subject to } Ax \leq b \\ & \quad \quad \quad x \geq 0 \end{aligned}$$

In his model,  $A$  is the coefficient matrix,  $x$  is the variable vector (all variables are non-negative),  $b$  is the right-hand-side vector, and  $c$  is the cost vector. John has solved his LP model and obtained an optimal solution  $x^*$ , that is,  $cx^* = c^*$  gives the lowest objective function value for his model. But he is wondering ... what is the "second best" solution (次佳, 亦即第二好的解) to his LP model?

Please help John to identify a second best solution to his model. To do so, you will modify John's LP model by adding new constraints, so that when the new model is solved, it will give the second best (lowest) objective function value for John's LP model. (請將 John 的 LP model 加入新的限制式。這個新增限制式之後的 LP model, 其最佳解就是 John 原始 LP model 的第二好解。) Note: you are not allowed to make any new definitions or assumptions for the new LP model—you can only use the information given in this problem to write the new LP model.

2. (15分) A production planning problem is to be solved using a mixed integer program (MIP). Two types of very expensive metals, Gold and Platinum, may be used. Let  $m_G$  be a binary (0/1) variable and define that if  $m_G = 1$ , it means that Gold will be used; if  $m_G = 0$ , it means that Gold will not be used. Similarly, let  $m_P$  be a binary variable and define that if  $m_P = 1$ , it means that Platinum will be used; if  $m_P = 0$ , it means that Platinum will not be used.
- a. (5分) Suppose " $m_G + m_P = 2$ " is a constraint in the MIP for solving the above production planning problem. What is the purpose of the constraint, that is, what will this constraint achieve (請寫出該限制式的作用, 亦即, 其意義為何)?
- b. (10分) Suppose " $m_G + m_P \leq 1$ " is a constraint in the MIP. Again, what is the purpose of this constraint?

3. (15分) Please solve the following linear program and find its optimal solution.

$$\begin{aligned} & \text{Minimize } 2x_1 + 3x_2 + 5x_3 + 2x_4 + 3x_5 \\ & \text{subject to } x_1 + x_2 + 2x_3 + x_4 + 3x_5 \geq 4 \\ & \quad \quad \quad 2x_1 - 2x_2 + 3x_3 + x_4 + x_5 \geq 3 \\ & \quad \quad \quad x_1, x_2, x_3, x_4, x_5 \geq 0 \end{aligned}$$

注意：背面有試題

參考用

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4. (15 points)

A postal service is modeled as an  $M/M/1$  queuing system with customer arrival rate of 2 per minute. It is desired to have fewer than 5 customers line up 99 percent of the time. How fast should the service rate be?

5. ( $10 * 2 = 20$  points)

A major traffic problem in Taipei involves traffic attempting to cross the Danshui River. The probability of no traffic delay in one period, given no traffic delay in the preceding period, is 0.85. The probability of finding a traffic delay in one period, given a delay in the preceding period, is 0.75. Traffic is classified as having either a delay or a no-delay state, and the period considered is 30 minutes.

- Assuming you are a motorist entering the traffic system and receive a radio report of a traffic delay, what is the probability that for the next 60 minutes (two time periods) the system will be in the delay state?
- What is the probability that in the long run the traffic will not be in the delay state?

6. (15 points)

The alarm system at a nuclear power plant is not completely reliable. If there is something wrong with the reactor, the probability that the alarm goes off is 0.99. On the other hand, the alarm goes off on 0.01 of the days when nothing is actually wrong. Suppose that something is wrong with the reactor only one day out of 100. What is the probability that something is actually wrong if the alarm goes off?

Let  $A =$  "something is wrong with reactor" and  $B =$  "alarm goes off"

參考用

注意：背面有試題