參考用

1. (10%) Find (a) $\int_a^b \dfrac{1}{\sqrt{1+x^2}}dx$,

    (b) $\int_a^b \dfrac{1}{\sqrt{x^2-1}}dx$, for $a,b>1$ or $a,b,<1$;

2. (10%) Find the volume of the solid by rotating the region bounded by the curves $xy=2, xy=4, x=1$ and $x=2$ about the y-axis.

3. (10%) Find the Taylor series at 0 for the following function: $f(x) = \dfrac{1}{\sqrt{1-x}} = (1-x)^{-1/2}$

4. (10%) Use the root test to find the radius of convergence of the following power series:
    $\sum_{n=1}^{\infty} \dfrac{n}{2^n} z^n$.

5. (10%) Find the following: (a) $\lim_{y\to 0} \log(1+y)/y$ (b) $\lim_{y\to\infty} y\log(1+1/y)$

6. Let $g: N \to N$ be a function such that $g(n+1) > g(n)$ for each n. Prove that for each $n \in N$, $g(n) \ge n$. (15 points)

7. Show that a sequence converges if and only if each of its subsequences converges. (20 points).

    Hint : From the definition of subsequences and result from Problem 6. Definition of subsequences: If $\{x_n\}$ is a sequence and $g: N \to N$ is a sequence such that $g(n+1) > g(n)$ for each $n \in N$ then $\{x_{g(n)}\}$ is a subsequence of $\{x_n\}$

8. Let $x_1 = \sqrt{6}$ and for $n>1$ let $x_n = \sqrt{x_{n-1}+6}$. Prove that $\{x_n\}$ converges and find the limits. (15 points)

    Hint: Need to show that $\{x_n\}$ is increasing