（1）　Suppose that Anderson always plays ping pong with Mark, Tom, and Wilson. Billy always plays with Zeus. Cora always plays with Mark and Zeus. Debbie always plays with Mark. If you want to tell each possible pair about tournament news with as few phone calls as possible, please model the above situation with a bipartite graph and construct the corresponding matrix first. Then find a minimum covering for the graph, and indicate the corresponding lines of the matrix. (10%)

（2）　Find a formula for s_n if s_n satisfies the recurrence relation
$s_n = 1.06\, s_{n-1} + 0.0216\, s_{n-2}$ for $n \geq 2$.
and the initial conditions $s_1 = 1100$ and $s_1 = 1166$. (10%)

（3）　Please use Karnaugh map method to simplify the expression of
$(x' \wedge y \wedge z) \vee (x \wedge y' \wedge z) \vee (x' \wedge y' \wedge z)$. (10%)

（4）　Consider the following network with the capacities shown on the edges. Please illustrate how to find a maximal flow and a minimal cut by starting with the flow that is 0 along every arc. Label the vertices in alphabetical order if choice of vertices has to be made.
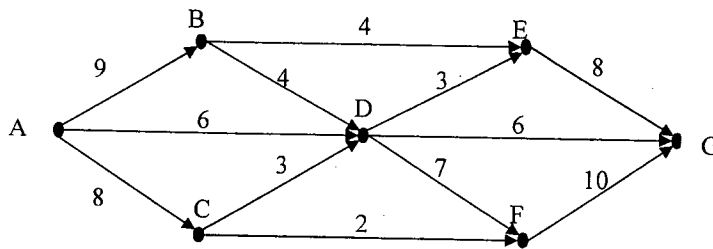
(10%)

（5）　Can you explain what does it mean for two rooted trees to be isomorphic? Besides, please draw a binary tree that generates the codewords 11, 00, 10, 010, 0110, 01111, 01110 at the terminal vertices. (10%)

（6）　(a) Please define *lexicographic order*. (4%)
(b) Consider the set $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Suppose that 6 distinct integers are selected from S. Please prove that at least two of the six integers have a sum equal to 11. (6%)

注意：背面有試題

（7）Consider the following state table with initial state u and accepting state w.

	u	v	w
0	v	w	w
1	u	u	v

Please draw the transition diagram for the finite state machine with this state table. Suppose that the finite state machine starts at the initial state, what is the state that the machine ends if the input string is 1011001? (10%)

（8）Suppose that $A = [I_k \mid J]$ is the $k \times n$ generator matrix of a (k, n)-code, where I_k is the $k \times k$ identity matrix and J is the $k \times (n-k)$ matrix whose columns are the last $n - k$ columns of A. Define the check matrix A^* associated with A by $A^* = \begin{bmatrix} J \\ \hline I_{n-k} \end{bmatrix}$.

(a) Determine the codeword for 1101 in the (4, 8)-code with the generator matrix

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}.$$

(4%)

(b) Please prove AA^* is a zero matrix based on the generator matrix A in (a). (6%)

（9）(a) How many 8-bit strings begin with 11 or end with 00? (5%)

(b) How many 8-bit strings begin with 1001 or 010? (5%)

（10）Suppose that $x R y$ means $x - y \in \{-4, 0, 4\}$. Prove that relation R is an equivalent relation on set $S = \{1, 2, 3, 4, 5, 6, 7, 8\}$. Besides, please describe the distinct equivalent classes of R. (10%)

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