

國立中央大學100學年度碩士班考試入學試題卷

所別：土木工程學系碩士班 結構組(一般生) 科目：工程數學 共 1 頁 第 1 頁

本科考試可使用計算器，廠牌、功能不拘

\*請在試卷答案卷(卡)內作答

參考用

- 1) The inverse Laplace transform of  $F(s) = \frac{5}{s^3 - 2s^2 + s - 2}$  can be denoted as  $f(t) = \mathcal{L}^{-1}\{F(s)\}$ . Please compute the value of  $f(t)$  at the time  $t = \pi$ . That is to find out the value of  $f(\pi)$ . (25%)
- 2) Let  $A = (A_{ij}) = \begin{pmatrix} -3 & 0 & -1 \\ -9 & 1 & 2 \\ -9 & 4 & -1 \end{pmatrix}$  be a  $3 \times 3$  matrix. If  $\lambda_1$ ,  $\lambda_2$  and  $\lambda_3$  are its eigen-values, please find out the sum of these three eigen-values. That is, please compute the value of  $\lambda_1 + \lambda_2 + \lambda_3$ . (25%)
- 3) A particular solution  $y(x)$  of the differential equation  $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + 9y = x$  satisfies the conditions  $y(1) = \frac{1}{10}$  and  $\frac{dy}{dx}(1) = \frac{1}{10}$ . That is,  $y = \frac{1}{10}$  and  $\frac{dy}{dx} = \frac{1}{10}$  at the point  $x = 1$ . Please find out the expression of  $y(x)$ . (25%)
- 4) There are two points  $A$  and  $B$  in the plane. The coordinates of  $A$  are  $(x, y) = (1, \frac{-\pi}{2})$  while those of  $B$  are  $(x, y) = (1, \frac{\pi}{2})$ . A path  $\Gamma$  connects  $A$  and  $B$ , and it is described by the equation  $x = (\sin y)^2$ . Compute the path integral  $J = \int_{\Gamma} Pdx + Qdy$  with  $P = \frac{1}{x^2y} - \frac{y}{x^2}$  and  $Q = \frac{1}{xy^2} + \frac{1}{x} + y$ . Note that the integration is carried out along  $\Gamma$  from point  $A$  to point  $B$ . (25%)