

國立中央大學100學年度碩士班考試入學試題卷

所別：地球物理研究所碩士班 不分組(一般生)

科目：微積分 共 2 頁 第 1 頁

本科考試禁用計算器

*請在試卷答案卷(卡)內作答

作答時須列出完整計算過程

1. (a) $\lim_{x \rightarrow 0} \frac{\sin 2x + \tan x}{3x} = ?$ [5%]

(b) $\lim_{x \rightarrow a} \frac{a-x}{\ln \frac{x}{a}} = ?$ [5%]

2. (a) $\frac{d}{dx} 2 \sin^{-1} \sqrt{1-2x} = ?$ [5%]

(b) $\frac{d}{dx} \sin(x^x) = ?$ [5%]

3. (a) $\int_{-\infty}^{\infty} e^{-x^2} dx = ?$ [5%]

(b) $\int_{-\pi}^{\pi} (\sin mx)(\sin nx) dx = ?$ (m, n 為整數) [5%]

4. Find the Fourier series of $f(x) = x^2$ ($-1 < x < 1$) and period $p = 2L = 2$. [10%]

5. 求由拋物線 $y = x^2$ 及直線 $x = 2$ 對 y 軸旋轉所圍成之迴轉體之體積。 [10%]

6. Find a general solution of the system $\begin{cases} \frac{dy_1}{dt} = -3y_1 + y_2 \\ \frac{dy_2}{dt} = y_1 - 3y_2 \end{cases}$ [10%]

7. Test for exactness. If exact, solve. If not, find an integrating factor and then solve.

(a) $9x dx + 4y dy = 0$ [5%]

(b) $-y dx + x dy = 0$ [5%]

參考用

注意：背面有試題

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8. Using Green's theorem to evaluate $\int_C \vec{F}(\vec{r}) \cdot d\vec{r}$ counterclockwise around the

boundary curve C of the region R , where $\vec{F} = [-y^3, x^3]$ and C the circle

$$x^2 + y^2 = 25 \quad [10\%]$$

9. Using separation of variables to solve the following partial differential equations [10%].

$$\frac{\partial^2 u(x, t)}{\partial t^2} = c^2 \frac{\partial^2 u(x, t)}{\partial x^2}, \quad u(0, t) = u(L, t) = 0,$$
$$u(x, 0) = f(x), \quad u_t(x, 0) = 0, \quad (0 \leq x \leq L).$$

10. 求(0, 0), (1, 1), (-1, -2)所決定的最佳直線 $y = ax + b$, 使得其誤差平方和最小 (the least squares solution)[10%].



注意：背面有試題