國立中央大學100學年度碩士班考試入學試題卷

所別：資訊工程學系碩士班 不分組(一般生)
資訊工程學系軟體工程碩士班 不分組(一般生)

科目：離散數學與線性代數　共 2 頁　第 1 頁

本科考試禁用計算器

＊請在試卷答案卷（卡）內作答

單一選擇題(每題答對給 2 分、答錯倒扣 0.5 分、不答 0 分)

1. V and W are finite dimensional vector spaces. Given $v_1, v_2 \in V$ and $w_1, w_2 \in W$, there exists a linear transformation $T: V \to W$ such that $T(v_1) = w_1$ and $T(v_2) = w_2$. (a) true. (b) false.
2. The row space and column space of any matrix A have the same dimension. (a) true. (b) false.
3. Suppose that the column vectors of matrix A are linearly independent. Then $A\mathbf{x} = \mathbf{b}$ has at most one solution for every $m\times 1$ matrix $\mathbf{b}$. (a) true. (b) false.
4. $v_1 = (2, 0, 2)$, $v_2 = (4, 0, 7)$, $v_3 = (-1, 1, 4)$ form a basis of R^3. (a) true. (b) false.
5. Let $v_1 = (0, 3, 9, 0)$, $v_2 = (1, -2, 0, 3)$, $v_3 = (2, -5, -3, 6)$, $v_4 = (2, -1, 4, -7)$, $v_5 = (10, -16, 2, 4)$. Suppose that V is the spaces spanned by these five vectors. Then, $\{v_2, v_3, v_4\}$ forms a basis for V. (a) true. (b) false.
6. Let $T: R^3 \to R^2$ be defined by $T(x, y, z)=(x+y, x-z)$. Then $T^{-1}(1,11)=(1,-1,1)*t+(11,-10, 0)$, where $t \in R$. (a) true. (b) false.
7. Let $T(p) = p'' - 2p' + 4p$, where p' and p'' are the first and the second derivative of polynomial p, respectively. Then T is a linear transformation on P_2, where P_2 is the set of all polynomials of degree at most (a) true. (b) false.
8. Let $V = M_{n\times n}(R)$ be a vector space. Then $W = \{A \in M_{n\times n} \mid A^{T} = A\}$ is a subspace of V. (a) true. (b) false.

Suppose that $T: R^2 \to R^2$ is linear and that $T(1, 0) = (1, 3)$ and $T(1, 1) = (2, 4)$.

9. rank$(T) = 2$. (a) true. (b) false.
10. nullity$(T) = 1$. (a) true. (b) false.

If $n\times n$ matrix A is diagonalizable, then

11. A has n distinct eigenvalues. (a) true. (b) false.
12. A^T is diagonalizable. (a) true. (b) false.
13. A has no zero eigenvalues. (a) true. (b) false.

If $\hat{x}$ is the least-squares solution of the inconsistent system $A\boldsymbol{x} = \boldsymbol{b}$, then

14. b is on Col$A$. (a) true. (b) false.
15. $(b - A\hat{x})$ is orthogonal to Col$A$. (a) true. (b) false.
16. sometimes $\hat{x}$ is not existed. (a) true. (b) false.

If $n\times n$ matrix A is a symmetric matrix, then

17. A^2 is also symmetric. (a) true. (b) false.
18. A is diagonalizable. (a) true. (b) false.
19. A has n positive eigenvalues. (a) true. (b) false.
20. A has n orthogonal eigenvectors. (a) true. (b) false.

多重選擇題 (每題答對給 5 分、答錯或不答 0 分)

21. Find a singular value decomposition $A = U\Sigma V^{T}$ with U and V being both orthogonal matrices, where $A = \begin{bmatrix} 2 & -1 \\ 2 & 2 \end{bmatrix}$. Which values are **not** in U or V matrices, (a) $1/\sqrt{2}$. (b) $1/\sqrt{3}$. (c) $1/\sqrt{5}$. (d) $2/\sqrt{3}$. (e) $-2/\sqrt{5}$.
22. Determine which of the following statements are true?
    (a) Given two bases for the same inner product space. There is always a transition matrix from one basis to the other basis
    (b) The transition matrix from B to B is always the identity matrix.
    (c) Any invertible $n\times n$ matrix is the transition matrix for some pair of bases for $\boldsymbol{R}^n$.
    (d) $\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$ is the transition matrix from $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ to $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$. (e) none of the above
23. Let $p(x)$, $q(x)$ represent the following predicates:
    "$p(x)$: x is even; $q(x)$: $x/2$ is even;" for the universe of all integers. Which of the following statements are logically equivalent to each other?
    (a) $\forall x(q(x) \to p(x))$.
    (b) If x is even, it is possible that $x/2$ is even too.
    (c) $\forall x$ ($q(x)$ is necessary for $p(x)$);
    (d) $\neg\exists x(\neg q(x) \wedge p(x))$
    (e) $\forall x(\neg p(x) \to \neg q(x))$.
24. Considering the Fibonacci numbers F_n that is recursively defined as: $F_{n+2}= F_{n+1} + F_n$; and $F_0 =0$; $F_1 =1$. Which of the following statements are true?
    (a) $F_n = \left(\frac{1+\sqrt{5}}{2}\right)^n + \left(\frac{1-\sqrt{5}}{2}\right)^n$
    (b) $F_{2n} = F_{n+1}^2 - F_{n-1}^2, n \geq 1$
    (c) Let $\mathbf{A} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$, $\mathbf{A}^n = \begin{bmatrix} F_{n+1} & F_n \\ F_n & F_{n-1} \end{bmatrix}, n \geq 0$
    (d) $\sum_{k=0}^{n} \binom{n}{k} F_k = F_{2n}, n \geq 0$
    (e) none of the above

注意：背面有試題

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25. To analyze the complexity of the following procedure P, We will use the following assumptions: Suppose P and Q are both procedures. Q take $\theta(m)$ time to compute, where m is the size of input; each statement line in and outside the loop counts 1 step.

Procedure P(array1[$a_1, a_2, \ldots, a_n$])

1. if n<4 exit.
2. declare initially new empty array2, array3;
3. for (i=1 to n)
4. { if ((i mod 4) =0)
5. insert a_i into array2;
6 if ((i mod 4) =3)
7. insert a_i into array3;
8. call Q(array1[$a_1, a_2, \ldots, a_n$]); }
9. call P(array2);
10. call P(array3);

Suppose n is a multiple of 4, which of the following relations on C_n can describe the time complexity of procedure P with respect to problem size n?

(a) $C_n = 4C_{n/3} + \theta(n)$　(b) $C_n = 2C_{n/4} + C_{n-1} + \theta(n)$　(c) $C_n = 2C_{n/4} + \frac{3}{2}\theta(n)$

(d) $C_n = 2C_{n/4} + \theta(n) + \theta(1)$　(e) none of the above

26. What can be the time complexity level of the procedure P in the question above?

(a) $O(n)$　(b) $O(n^2 \log n)$　(c) $O(n^2)$　(d) $O(n^{\log_4^2})$ $\log_4 2$　(e) none of the above

27. To solve the recurrence relation $a_{n+2} = 5a_{n+1} - 6a_n + 2,\ n \ge 0, a_0 = 3, a_1 = 7.$, what of the followings will be the corresponding generating function $f(x)$?

(a) $f(x) = (3-7x)/(1-5x+6x^2)$.　(b) $f(x) = (3-5x)/(1-4x+3x^2)$.

(c) $f(x) = \frac{2}{1-3x} + \frac{1}{1-x}$.　(d) $f(x) = \frac{2}{1-3x} + \frac{1}{1-2x}$.

(e) none of the above.

28. Draw a graph with 64 vertices representing the squares of a chessboard. Connect two vertices with an edge if you can move legally between the corresponding squares with a single move of a knight. [The moves of a knight are L-shaped, two squares vertically (or horizontally) followed by one square horizontally (respectively, vertically).]

(a) This graph is bipartite.　(b) Four vertices of degree 2.

(c) Four vertices of degree 3.　(d) Twenty vertices of degree 6.

(e) Sixteen vertices of degree 8.

29. Which of the following statements are true?

(a) In a Hamiltonian graph, every edge belongs to some Hamiltonian cycle.

(b) In a Hamiltonian graph, every edge belongs to a cycle.

(c) Every Eulerian graph contains a subgraph that is Hamiltonian.

(d) Every Hamiltonian graph contains a subgraph that is Eulerian.

(e) Suppose G_1 and G_2 are isomorphic graphs. Either both G_1 and G_2 are connected or else neither is connected.

30. Define $g: A \to B$ by $g(x) = x^3 - x + 1$.

(a) A = R, B = R, g is onto but not one-to-one.　(b) A = Z, B = Z, g is onto but not one-to-one.

(c) A = Z, B = Z, g is neither onto nor one-to-one.　(d) A = N, B = N, g is onto but not one-to-one.

(e) A = N, B = N, g is neither onto nor one-to-one.

31. Determine which of the following relations (~) define equivalence relations on the set A.

(a) A is the set of all triangles in the plane; for a, b ∈ A, a ~ b if and only if a and b are congruent.

(b) A is the set of all circles in the plane; for a, b ∈ A, a ~ b if and only if a and b have the same center.

(c) A is the set of all straight lines in the plane; for a, b ∈ A, a ~ b if and only if a is parallel to b.

(d) A is the set of all lines in the plane; for a, b ∈ A, a~ b if and only if a is perpendicular to b.

(e) A is the set of points different from the origin in the Euclidean plane. for a, b ∈ A, a ~ b if a = b or the line through the distinct points a and b passes through the origin.

32. In a room where there are more than 50 people with ages between 1 and 100.

(a) Either two people have the same age or there are two people whose ages are consecutive integers.

(b) Either two people have the same age or one person's age is a multiple of another's.

(c) Some of the people shake hands. At least two shook the same number of hands. ("No hands" is a possibility)

(d) Some people shake hands. Among those who shook at least one hand, two people shook the same number of hands.

(e) None of the above.

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