系所組別： 工程科學系甲、乙、丙、丁、戊、己組

考試科目： 工程數學

※ 考生請注意：本試題 □可 ☒不可 使用計算機

1. Solve the problem $\frac{d^2x}{dt^2}+3\frac{dx}{dt}+2x=f(t)$, $x(0)=\frac{dx}{dt}(0)=0$ for the following two different cases:

   (a) $f(t)=t$, $-1\le t<1$, is a periodic function. (18%)

   (b) $f(t)$ is a general function. (10%)

2. Solve the problem $\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}=0$, $1\le r\le 2$, $0\le\theta\le 60^0$, with the boundary conditions

   $T(1,\theta)=0$, $T(2,\theta)=g(\theta)$, $\frac{\partial}{\partial\theta}T(r,0)=\frac{\partial}{\partial\theta}T(r,60^0)=0$. (25%)

3. (a) Find all real values of k for which the matrix

$$A=\begin{bmatrix}1 & k & 3\\ -k & 2 & -k\\ 1 & k & 3\end{bmatrix}$$

   has real eigenvalues. (10%)

   (b) With k real, determine the largest possible eigenvalue of A and the corresponding eigenvector. (7%)

   (c) Find the real values of k for which A has only two real eigenvalues. Find the two eigenvalues and the corresponding eigenvectors. (10%)

4. (a) Calculate $\int_0^{\pi}\frac{1}{\alpha+\beta\cos\theta}d\theta$, $\alpha>\beta>1$. (10%)

   (b) Using the result of part (a) to calculate $\int_0^{\pi}\frac{1}{(\alpha+\beta\cos\theta)^3}d\theta$. (10%)