系所組別： 光電科學與工程研究所甲、乙組

考試科目： 工程數學

考試日期：0219，節次：3

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1. The first six Legendre polynomials are

$P_0(x) = 1$, $P_1(x) = x$

$P_2(x) = \frac{1}{2}(3x^2 - 1)$, $P_3(x) = \frac{1}{2}(5x^3 - 3x)$

$P_4(x) = \frac{1}{8}(35x^4 - 30x^2 + 3)$, $P_5(x) = \frac{1}{8}(63x^5 - 70x^3 + 15x)$

Find the first three positive values of λ for which the problem

$(1-x^2)y'' - 2xy' + \lambda y = 0$

$y(0) = 0$, $y(x)$, $y'(x)$ bounded on [-1,1]

has nontrivial solutions (solutions other than $y(x)=0$). (10%)

2. The square error of a function F relative to a function f on the interval $-\pi \le x \le \pi$ is defined as

$$E = \int_{-\pi}^{\pi} (f - F)^2 dx$$

Suppose we want to minimize the square error of a function $F=a+b\sin x$ (a and b are constants) relative to $f(x)=x+\pi$ ($-\pi < x < \pi$), what are the best choices of constants a and b that give the smallest square error? (10%)

3. Solve $2y'' + ty' - 2y = 10, y(0) = y'(0) = 0$

(a) Find the Laplace transform of the differential equation.

(b) Solve the 1st-order equation from (a).

(c) Solve $y(t)$ by finding the inverse Laplace transform of the solution in (b).

(10%)

4. Solve

$$\frac{1}{2}\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, \quad x > 0, \ t > 0$$

$$\left.\frac{\partial u}{\partial x}\right|_{x=0} = 0, \quad t > 0$$

$$u(x,0) = e^{-2x}, \quad x > 0$$

(10%)

5. Find the eigenvalues and eigenfunctions of the boundary value problem

$y'' + \lambda y = 0, \quad y'(0) = 0, y'(L) = 0$ (10%)

(背面仍有題目，請繼續作答)

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6. The set B = $\{u_1, u_2, u_3\}$, where
$u_1 = \langle 1,1,1\rangle, u_2 = \langle 1,2,2\rangle, u_3 = \langle 1,1,0\rangle$
is the basis for R^3. Transform B into an orthonormal basis B". (10%)

7. Use the inverse of the matrix **A** to solve the system **AX=B**,
where $\mathbf{A} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 0 \\ 0 & 1 & 2 \end{pmatrix}$, and the vector **B** is given by $\begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix}$ (10%)

8. Evaluate the integral $\iint_R \left(\sqrt{(x-y)^2 + 2(x+y)+1}\right)^{-1} dA$
where R is the region bounded by the graphs of $y = x$, $x = 2$, and $y = 0$ by means of the change of variables $x = u + uv, y = v + uv$. (10%)

9. Expand $f(z) = \frac{2}{z^2-4z+3}$ in an appropriate series valid for
(a) $|z| > 3$; (b) $0 < |z-1| < 2$ (10%)

10. Let $f(z) = z^n g(z)$, where n is a positive integer, $g(z)$ is entire, and $g(z) \neq 0$ for all z. Let C be a circle with center at the origin.
Evaluate $\oint_C \frac{f'(z)}{f(z)} dz$ (10%)