

系所組別： 物理學系

考試科目： 古典力學

考試日期： 0219，節次： 1

※ 考生請注意：本試題 可 不可 使用計算機

1. (25 %) A particle of mass m moving in one dimension has potential energy $U(x) = U_0[2(x/a)^2 - (x/a)^4]$, where U_0 and a are positive constants.
 - (a) Find the force $F(x)$, which acts on the particle. (3 %)
 - (b) Sketch $U(x)$. Find the position of stable and unstable equilibrium. (4 %)
 - (c) What is the angular frequency ω of oscillations about the point of stable equilibrium? (5 %)
 - (d) What is the minimum speed the particle must have at the origin to escape to infinity? (5 %)
 - (e) At $t = 0$ the particle is at the origin and its velocity is positive and equal in magnitude to the escape speed of part (d). Find $x(t)$. (8 %)
2. (15 %) An undamped driven harmonic oscillator satisfies the equation of motion $m(d^2x/dt^2 + \omega_0^2x) = F(t)$. The driving force $F(t) = F_0 \sin(\omega t)$ is switched on at $t = 0$.
 - (a) Find $x(t)$ for the initial conditions $x = 0$ and $v = 0$ at $t = 0$. (10 %)
 - (b) Find $x(t)$ for $\omega = \omega_0$ by taking the limit $\omega \rightarrow \omega_0$ in your result in part (a). (5 %)
3. (20 %) A particle of mass m is attracted to a force center with the force with magnitude k/r^2 . Use plane polar coordinates and find the Hamiltonian's equations of motion.
4. (15 %) Find the horizontal deflection from the plumb line caused by the Coriolis force acting on a particle falling freely in Earth's gravitational field from a height h above Earth's surface.
5. (25 %) A string is pulled aside a distance h at a point $3L/7$ from one end. At a point $3L/7$ from the other end, the string is pulled aside a distance h in the opposite direction. Discuss the vibrations in terms of normal modes.