考 試 科 目	計算機數學	所 別	8141 資訊科學系	考試時間	2 月 25 日(六) 第 三 節

I 離散數學部分(60 %)

(1) 是非題 (10%)

1.1. $f(n) = O(n^2)$ implies $f(n) = O(n^3)$.

1.2. If A ,B and C are sets of strings, then $(A \cap B)$ C= AB $\cap$AC.

1.3.The logical expressions $\sim(p \rightarrow q)$ and $\sim p \rightarrow \sim q$ are logically equivalent.

1.4. Every complete lattice is bounded.

1.5. There are regular languages which are not context-free.

1.6. If G is a simple graph with 10 vertices and 35 edges, it must be connected.

1.7. There exists a Eular circuit in a complete simple graph K_6 with 6 vertices.

1.8. A binary tree of height $n \geq 0$ has no more than 2^n leaves.

1.9. A tree has more vertices than edges.

1.10. If G is a digraph with 20 vertices and has a path of length $k \geq 20$, then it must have also a path of length 133.

(2) 填充題(20%;每格 2%)：

2.1. If we randomly construct a simple graph G with 5 vertices. Let Pr be the probability of G having more than 5 edges. If Pr = q/p where p and q are non-negative integers and gcd(p, q) = 1. Then p+q = ________.

2.2. There are ______ different Boolean functions $f(x_1,\ldots,x_n)$ of degree $n > 0$, among which there are _____ different Boolean functions satisfying the condition that $f(x_1,\ldots,x_n) = f(y_1,\ldots,y_n)$ if $x_1+\ldots+x_n = y_1+\ldots+y_n$.

2.3. The rooted Fibonacci tree T_n are defined recursively in the following way: T_1 and T_2 are both rooted tree consisting of a single vertex, and for $n \geq 3$, the tree T_n is constructed from a root with T_{n-1} as its left subtree and T_{n-2} as its right subtree. Then T_{10} has ______ vertices and its height is _________.

2.4. Let f be an increasing function satisfies the divide-and-conquer relation $f(n) = 5\, f(n/2) + 2n^2$ and the initial condition $f(1) = 1$. Then f(n) has the asymptotic order $\Theta($_______________$)$.

2..5 The equation $x + y + z < 20$ has _________ non-negative solutions.

2.6. There are ____ ways to assign 5 different jobs to 4 different employees if every employee is assigned at least one jobs.

2.7. Let M_R given below be the matrix representation of a relation R. Find the matrix $M_{R \cdot R}$ representing R·R and the matrix M_{R+} representing the transitive closure R+ of R.

$$M_R = \begin{bmatrix} 101 \\ 110 \\ 010 \end{bmatrix}, \quad M_{R \cdot R} = \_\_\_\_\_\_\_\_\_\_\_\_, \text{ and } M_{R+} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_.$$

備 註	試 題 隨 卷 繳 交

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I 離散數學部分(續前頁)

計算與證明: (30%)

(3). [10%] Solve the following recurrence relation:

$$a_n = 2\, a_{n-1} - 2\, a_{n-2}$$

with the initial conditions that $a_0 = 1$ and $a_1 = 2$. Note your solution expression may contain complex numbers.

(4). [10%] Prove by induction that for all integer $n \geq 0$, the number $f_n =$

$$\frac{5+\sqrt{5}}{10}\left(\frac{1+\sqrt{5}}{2}\right)^n + \frac{5-\sqrt{5}}{10}\left(\frac{1-\sqrt{5}}{2}\right)^n$$

is a non-negative integer.

(5) [10%] A poset $(S, \leq)$ is said to be well-founded if there is no infinite sequence of elements $x_1, x_2, x_3, \ldots$ such that $x_1 > x_2 > x_3 \ldots$, where $x_i > x_{i+1}$ means $x_{i+1} \leq x_i$ but $x_i \neq x_{i+1}$. An equivalent definition of well-foundedness is called the minimal condition which the poset $(S, \leq)$ is said to satisfy if every nonempty subset of S has a minimal element. Show that both definitions are indeed equivalent by completing the proof of the following propositions:

(a) if $(S, \leq)$ in not well-founded, then it does not satisfy the minimal condition. [4%]

(b) if $(S, \leq)$ does not satisfy the minimal condition, then it is not well-founded. [6%]

備註	試題隨卷繳交

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II. 線性代數(40%)

(10%) 1. Let

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 3 & k \\ 1 & k & 3 \end{bmatrix}$$

Find the values of k for which det $A = 0$ and hence, or otherwise, determine the value of k for which the following system has more than one solution:

$$x + y - z = 1$$

$$2x + 3y + kz = 3$$

$$x + ky + 3z = 2.$$

Solve the system for this value of k and determine the solution for which $x^2 + y^2 + z^2$ has least value.

(10%) 2. Let

$$U = \left[\begin{array}{cc|ccc} 1 & 2 & 0 & 0 & 0 \\ 3 & 4 & 0 & 0 & 0 \\ \hline 0 & 0 & 5 & 1 & 2 \\ 0 & 0 & 3 & 4 & 1 \end{array}\right] \text{ and } V = \left[\begin{array}{cc|cc} 3 & -2 & 0 & 0 \\ 2 & 4 & 0 & 0 \\ \hline 0 & 0 & 1 & 2 \\ 0 & 0 & 2 & -3 \\ 0 & 0 & -4 & 1 \end{array}\right]$$

(a) Find UV using block multiplication (b) Are U and V block diagonal matrices? (c) Is UV block diagonal?

(10%) 3. Determine whether or not W is a subspace of R^3 where W consists of all vectors (a, b, c) in R^3 such that:

(a) $a = 3b$ (b) $a \le b \le c$ (c) $ab = 0$

(d) $a + b + c = 0$ (e) $b = a^2$ (f) $a = 2b = 3c$

(10%) 4. Suppose $v = (1, 3, 5, 7)$. Find the projection of v onto W or, in other words, find $w \in W$ that minimizes $\| v - w \|$, where W is the subspace of R^4 spanned by:

(a) $u_1 = (1, 1, 1, 1)$ and $u_2 = (1, -3, 4, -2)$

(b) $v_1 = (1, 1, 1, 1)$ and $v_2 = (1, 2, 3, 2)$

備註	試題隨卷繳交