東吳大學 105 學年度碩士班研究生招生考試試題

系級	數學系碩士班 A 組(數學)	考試時間	100 分鐘
科目	高等微積分	本科總分	100 分

1.(20%) Let $f(x)=\begin{cases} x & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$. Show that f is continuous at $x=0$ and nowhere else.

2. (15%) Let $f(x,y,z)=y^3+\ln(x+z^2)$

(1) Find $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$, and $\frac{\partial f}{\partial z}$.

(2) Explain that f is differentiable at each point of $D=\{(x,y,z)\in R^3 : x>0\}$.

(3)Use the differential to estimate the difference $f(1.1,1.2,-0.1)-f(1,1,0)$

3 (15%) If f is continuous on $[a,b]$ and $\int_a^b |f(x)|dx=0$, then $f(x)=0$ for all $x\in[a,b]$.

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4.(15%) Let $f(x)=\begin{cases} e^{-1/x^2} & \text{if } x\neq 0 \\ 0 & \text{if } x=0 \end{cases}$. Prove that $f'(0)=0$.

5. (15%) Suppose f is differentiable on an open set $S\subset R^n$ and has a local maximum at $\vec{x}_0\in S$.

Prove that $\nabla f(\vec{x}_0)=\vec{0}$.

6. (20%) Suppose that $\varnothing\neq E\subset R$ and that $f_n\to f$ uniformly on E. Prove that if each f_n is uniformly continuous on E, then f is uniformly continuous on E.