國立中央大學105學年度碩士班考試入學試題

所別： 水文與海洋科學研究所 碩士班 不分組(一般生)　　　　共1頁　第1頁

科目： 流體力學

本科考試禁用計算器　　　　*請在答案卷(卡)內作答

1. (20%) Given the velocity field

   $\vec{v} = 10\vec{i} + \left(x^2 + y^2\right)\vec{j} - 2xy\vec{k}$　　[m/s]

   What is the acceleration of a particle at position (3,1,0) m?

   Hint: the acceleration can be calculated as:

   $$\vec{a} = \frac{\partial \vec{v}}{\partial t} + \left(\vec{v} \cdot \nabla\right)\vec{v} = \frac{\partial \vec{v}}{\partial t} + v_x \frac{\partial \vec{v}}{\partial x} + v_y \frac{\partial \vec{v}}{\partial y} + v_z \frac{\partial \vec{v}}{\partial z}$$

2. The velocity potential of a steady flow is given by the equation:

   $\phi = x^2 + y^2 - 2z^2$

   (1) (10%) Find the velocity components $\left(u, v, w\right)$ for the flow field as

   $u = \partial\phi / \partial x, v = \partial\phi / \partial y, w = \partial\phi / \partial z$

   (2) (10%) Show that this field represents a possible incompressible, irrotational flow.

   (3) (20%) The temperature of the field is described by the following expression:

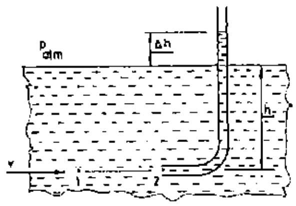
   $T = x + xy + z^2 + 2xyz$

   Based on the chain rule,

   $$\frac{DT}{Dt} = \frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z},$$

   please determine the time rate of change of the temperature of a fluid element as it passes through the point $\left(1, -2, 3\right)$.

3. (20%) A glass tube with a 90° bend is open at both ends. It is inserted into a flowing stream of liquid so that one opening is directed upstream and the other is directed upward. Liquid inside the tube is 0.05m higher than the surface of the flowing liquid. Determine the velocity measured by the tube. You may assume the gravitational acceleration g = 10 m/s².



4. (20%) The 2-D stream function for a flow is

   $\Psi = 9 + 6x - 4y + 7xy$

   Find the velocity potential.

   Hint:

   $$v_x = \frac{\partial \Psi}{\partial y} = \frac{\partial \Phi}{\partial x}, v_y = -\frac{\partial \Psi}{\partial x} = \frac{\partial \Phi}{\partial y}$$