

國立中央大學 105 學年度碩士班考試入學試題

所別： 統計研究所 碩士班 不分組(一般生)  
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科目： 數理統計

本科考試可使用計算器，廠牌、功能不拘

\*請在答案卷(卡)內作答

1. Find the constant  $c$  such that the following functions are p.d.f.s.

(a)  $c \cdot e^{-\frac{(x-3)^2}{12}}, x \in R$  (10%)

(b)  $c \cdot x^6 e^{-3x}, x > 0$  (10%)

2. Let  $X_1$  and  $X_2$  have the joint p.d.f.  $f(x_1, x_2)$  described as follows:

$(x_1, x_2)$	(1,1)	(1,2)	(1,3)	(2,1)	(2,2)	(2,3)
$f(x_1, x_2)$	2/15	4/15	3/15	1/15	1/15	4/15

and  $f(x_1, x_2)$  is equal to zero elsewhere.

(a) Find the marginal probability density functions for  $X_1$  and  $X_2$ . (10%)

(b) Find the conditional mean of  $X_1$  given  $X_2 = x_2$ . (10%)

3. Let  $f(x, y) = 2, 0 < x < y, 0 < y < 1$ , be the joint pdf of  $X$  and  $Y$ . Obtain

(a) the correlation coefficient of  $X$  and  $Y$ , (10%)

(b)  $Var(X|Y = y)$ . (10%)

4. Let  $X_1, \dots, X_n$  be a random sample from the following p.d.f.s:

(a).  $f(x; \theta) = (1/\theta)e^{-x/\theta}, 0 < x < \infty, 0 < \theta < \infty$ . (10%)

(b).  $f(x; \theta) = e^{-(x-\theta)}, \theta \leq x < \infty, -\infty < \theta < \infty$ . (10%)

In each case find the Maximum likelihood estimator of  $\theta$ .

5. Let  $X_1$  and  $X_2$  be independent random variables distributed as exponential with parameter  $\lambda=1$ . The p.d.f.  $f(x) = \exp(-x), x > 0$ .

(a) Derive the p.d.f. of  $X_1 + X_2$  and  $X_1 / X_2$ , respectively. (10%)

(b) Show that  $X_1 + X_2$  and  $X_1 / X_2$  are independent. (10%)