

考試科目	微積分 81111 81161	所別	應用數學系	考試時間	2 月 28 日(日) 第一節
------	--------------------	----	-------	------	-----------------

※ Show all your work for full credit. Unjustified answers will receive no credit.

- Let $f: R \rightarrow R$ be a function satisfying $f(x+y) = f(x) + f(y)$ for all $x, y \in R$. Show that:
 - $f(0) = 0$ and $f(-x) = -f(x)$ for all $x \in R$.
 - if, in addition, f is continuous at 0, then f is continuous on R . (20%)
- Let $ax + by + cz + d = 0$ be a plane in the space R^3 . Use Lagrange multipliers to show that the shortest distance from a point (x_0, y_0, z_0) in R^3 to the plane is $\frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}}$. (20%)
- Let

$$f(x) = \begin{cases} \frac{\sin x}{x}, & 0 < x \leq \pi, \\ 1, & x = 0, \end{cases}$$
 and R be the region in the plane R^2 bounded by the curve $y = f(x)$, x -axis, and y -axis. Find the volume of the solid generated by revolving the region R about the y -axis. (20%)
- Let R be a region in the plane R^2 bounded by a simple closed curve C .
 - Show that the area of R is given by $\frac{1}{2} \oint_C xdy - ydx$.
 - Use (a) to compute the area of the region bounded by the ellipse $\frac{x^2}{4^2} + \frac{y^2}{5^2} = 1$. (20%)
- Let $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ be convergent series.
 - Is $\sum_{n=1}^{\infty} a_n b_n$ convergent? Justify your answer.
 - If, in addition, $a_n \geq 0$ and $b_n \geq 0$ for all $n \in N$, show that $\sum_{n=1}^{\infty} a_n^2$ and $\sum_{n=1}^{\infty} a_n b_n$ both converge. (20%)

備註 一、作答於試題上者，不予計分。
二、試題請隨卷繳交。