考 試 科 目 微積分 P4212A 所 別 企管研究所乙組 考試時間 2月27日（六）第三節

※ **Show all calculations and display answers clearly. Unjustified answers will receive no credit.**

1. (a) Find $\frac{dy}{dx}$, where $y = (\ln x)^{\ln x}$.

   (b) Evaluate the limit: $\lim_{x \to 0+} (1 + 5x)^{2/x}$. (10%).

2. Let $x = t^5 + 10t^2 + 5t + 2$ and $y = \dfrac{x}{x^2 + 1}$. Find $\dfrac{dy}{dt}\Big|_{t=0}$ and $\dfrac{d^2 y}{dt^2}\Big|_{t=0}$. (10%)

3. Using Lagrange multipliers, find the distance from the point A(1,2,3) to the point on the plane x − 2y + 2z =6 closest to A. (10%)

4. The temperature x miles east and y miles north of a weather station is given by the function $f(x, y) = 24 + 2x - y$. Find the average temperature over the region $R = \{(x, y) | 0 \le x \le y, 0 \le y \le 2\}$. (10%)

5. A company's profit in dollars from producing x tape decks and y CD player per day is $P(x, y) = 3x^2 - xy + 2y^2$. If the company currently produces 200 tape decks and 500 CD players, use the marginal profit function to estimate the change in profit that would result from producing one more CD player. (10%)

6. A company manufactures two products. Product A costs \$35 per unit to manufacture and product B costs \$55 per unit to manufacture. Marketing research indicates that if the company makes x product A and y product B per month and wants to sell them all, they will have to sell product A at a price of $150 - \frac{1}{2}x + \frac{1}{2}y$ dollars per unit and product B at a price of $225 + \frac{1}{2}x - y$ dollars per unit. Find the quantities and the prices of the two products that maximize profit. (10%)

備 註 一、作答於試題上者，不予計分。
二、試題請隨卷繳交。

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7. A manufacturer produces erasers. The fixed cost is \$12000 per week. The marginal cost for producing x erasers per week is $5 + \frac{x}{5000}$ dollars. According to marketing research, the demand function is $D(p) = 200000 - 10000p$, $0 \le p \le 20$. Here p is the selling price in dollars and $D(p)$ is the number of erasers that will be sold per week at price p. How many erasers should they make and sell per week to maximize their profit? (10%)

8. (a) The function $f(x)$ is defined by

$$f(x) = \sum_{n=0}^{\infty} (n+1)2^{n+1} x^n$$

   Determine the radius of convergence.

   (b) Write the power series for $g(x) = \int_0^x f(t)dt$ and compute the sum of the series.
   (c) Compute the sum of the series in (a). (15%)

9. Evaluate the following integrals.

   (a) $\int_1^4 \frac{1}{\sqrt{x}(\sqrt{x}+1)^2} dx$ 　(b) $\int_0^1 xe^x dx$ 　(c) $\int \frac{1}{x^2-1} dx$ (15%)

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