

※ 考生請注意：本試題 可 不可 使用計算機 請勿在本試題紙上作答，否則不予計分

Please leave a space and summarize your answer in the beginning of your answer sheet according to the format shown as follows.

1.			
2			
3	(a)	(b)	
4	(a)	(b)	(c) (d)
5			
6	(a)	(b)	
7			
8			
9	(a)	(b)	

- (10%) Suppose there are three chests each having two drawers. The first chest has a gold coin in each drawer, the second chest has a gold coin in one drawer and a silver coin the other drawer, and the third chest has a silver coin in each drawer. A chest is chosen at random and a drawer opened. If the drawer contains a gold coin, what is the probability that the other drawer contains a gold coin?
- (10%) Suppose n balls are identically and independently distributed into n boxes. Compute the probability that only box 1 is empty.
- (10%) In present day, toys are placed in candy boxes to entice young purchasers. Suppose that there are r different types of toys, and that a given box is equally likely to contain any one of them. If n boxes are purchased, find the probability of
 - having collected at least one of each type?
 - of missing exactly k of the r types?
- (20%) Let X and Y be independent random variables each geometrically distributed with parameter p .
 - Find the distribution of $\min(X, Y)$
 - Find the probability $P(\min(X, Y)=X)$
 - Find the distribution of $X+Y$
 - Find the probability $P(Y=y|X+Y=z)$ for $y=0, 1, \dots, z$.

(背面仍有題目,請繼續作答)

系所組別：製造資訊與系統研究所乙組

考試科目：機率與統計

考試日期：0220，節次：3

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5. (10%) Let T_i be the number of trials up to and including the i -th success.
Let $0 \leq x_1 < \dots < x_r$ be integers. Compute the probability $P(T_1=x_1, T_2=x_2, \dots, T_r=x_r)$.
6. (10%) Let X_1, X_2, X_3 be independent random variables each uniformly distributed on $(0,1)$.
(a) Find the density of the random variable $Y = X_1 + X_2 + X_3$.
(b) Find the probability $P(X_1 + X_2 + X_3 \leq 2)$.
7. (10%) A polling organization samples 1200 voters to estimate the proportion planning to vote for candidate A in a certain election. How large would the true proportion p have to be for candidate A to be 95% sure that the majority of those sampled will vote for him?
8. (10%) Suppose the length of life of a certain kind of light bulb, after it is installed, is exponentially distributed with a mean length of 10 days. As soon as one light bulb burns out, a similar one is installed in its place. Find the probability that more than 50 bulbs will be required during a one-year period.
9. (10%) Two players having respective initial capitals of \$5 and \$10 agree to make a series of \$1 bets until one of them goes broke. Assume the outcomes of the bets are independent and both players have probability $1/2$ of winning any given bet.
(a) Find the probability that the player with the initial capital of \$10 goes broke
(b) Find the expected number of bets.

Reference:

$$\Phi(z) = \int_{-\infty}^z \frac{1}{\sqrt{2\pi}} e^{-u^2/2} du = P(Z \leq z)$$

$\Phi(1.10) = 0.8643$	$\Phi(-1.90) = 0.0287$	
$\Phi(1.11) = 0.8665$	$\Phi(-1.91) = 0.0281$	$\Phi(-1.64) = 0.0505$
$\Phi(1.12) = 0.8686$	$\Phi(-1.92) = 0.0274$	$\Phi(-1.65) = 0.0495$
$\Phi(1.13) = 0.8708$	$\Phi(-1.93) = 0.0268$	