國立政治大學 101 學年度碩士班招生考試試題

考試科目	微積分	所別	8111, 8116 應用數學系	考試時間	2月25日(六) 第一節

1. Use Lagrange multipliers to find the points on the ellipsoid
$$x^2 + y^2 + z^2 - 3y + yz - 6z + 5 = 0$$
that are closest and farthest from the xy-plane. (20%)

2. Let $f(x,y) = \begin{cases} \frac{xy}{x^2+y^2}, & \text{if } (x,y) \neq (0,0) \\ 0, & \text{if } (x,y) = (0,0). \end{cases}$

Show that $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ both exist on $\mathbb{R}^2$, but $f(x,y)$ is not continuous on $\mathbb{R}^2$. (20%)

3. Let C be the ellipse $\frac{x^2}{4^2} + \frac{y^2}{3^2} = 1$. Compute the line integral
$$\frac{1}{2}\oint_C x\,dy - y\,dx.$$ (20%)

4. Show that the sequence $\{a_n\}$ and the series $\sum_{n=1}^{\infty}(a_{n+1} - a_n)$ both converge of both diverge. (20%)

5. Let $\{a_n\}$ be a sequence in $\mathbb{R}$ and $\lim_{n\to\infty} a_n = A$ exist. Show that
$$\lim_{n\to\infty} \frac{a_1 + \cdots + a_n}{n} = A.$$ (20%)

備註	試題隨卷繳交