

系所組別： 統計學系

考試科目： 數學

考試日期： 0220，節次： 1

※ 考生請注意：本試題 可 不可 使用計算機

1. Find the following derivatives. (10%)

(a) $\frac{d}{dx} e^{\sin^2 x}$ at $x = \frac{\pi}{2}$

(b) $D_x \cos(\ln |2x|)$ at $x = -\frac{1}{2}$

2. Find the following integrals. (20%)

(a) $\int_{\ln 2}^{\ln 3} \frac{e^{-x}}{\sqrt{1-e^{-2x}}} dx$

(b) $\int_0^3 x \ln \sqrt{1+x^2} dx$

(c) $\int_0^\infty x^2 e^{-x^2} dx$

(d) $\int_{-\infty}^\infty x^3 e^{-|x|} dx$

3. Find the following limits: (20%)

(a) $\lim_{n \rightarrow \infty} n(3^{1/n} - 1)$

(b) $\lim_{t \rightarrow 0} \frac{1}{t} \int_0^t \sqrt{1+\cos^2 u} du$

(c) $\lim_{x \rightarrow 0} \frac{\cos x - \cos 3x}{\sin(x^2)}$

(d) $\lim_{x \rightarrow \infty} \frac{1}{x} \int_0^x e^{t^2} dt$

4. Find the Taylor series expansion of $f(x) = \sqrt{x} \tan^{-1} \sqrt{x}$ in powers of x . (10%)5. Find the volume of the solid bounded above by the cone $z = 2 - \sqrt{x^2 + y^2}$ and below by the disk $\Omega: (x-1)^2 + y^2 \leq 1$. (10%)6. Suppose the product of A and B is the zero matrix: $AB = 0$. (10%)(a) Then the _____ space of A contains the _____ space of B . (Why?)(b) Suppose the matrix A is m by n with rank r and the matrix B is n by k with rank s . Prove that $r + s \leq n$.7. Suppose a 3×3 matrix A has eigenvalues 0, 2, 4 with eigenvectors u, v, w . (10%)(a) Are u, v, w independent? Why? (3%)(b) Find a particular solution to $Ax = v + w$. Find all solutions. (4%)(c) Does $Ax = u$ have a solution? Why? (3%)8. Factor $A = \begin{bmatrix} 1 & 2 \\ ? & 3 \end{bmatrix}$ into $A = SAS^{-1}$. Write down S, Λ, S^{-1} exactly.What is the matrix of $S(\Lambda + 2I)^3 S^{-1}$? (10%)