招生學年度	105	招生類別	碩士班
系所班別	運籌管理研究所碩士班（工業工程與決策科學組）		
科目名稱	作業研究		
注意事項	本考科可使用掌上型計算機		

There are 5 questions in this examination paper. Points of parts are shown there. Show your steps and argument for all parts.

考卷附錄列出部份題目之中譯，但一切以英文題目為本。

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1. Solve the following minimization problems.

(a). (3 points) Min $(x-6)^2$.

(b). (3 points) Min $(x-6)^2$, *s.t.* $x \le 3$.

(c). (3 points) Min $(x-6)^2 + (y-10)^2$.

(d). (3 points) Min $(x-6)^2 + (y-10)^2$, *s.t.* $x + y = 8$.

(e). (3 points) Min $(x-6)^2 + (y-10)^2$, *s.t.* $x + y \le 8$.

2. The constraints of a linear program are: $x - y \ge 0, x \le 5, x \ge 0, y \ge 0$.

(a). (5 points) Sketch the feasible region of the linear program.

(b). (5 points) Find the co-ordinates of the three corner points (i.e., extreme points) in the feasible region.

(c). (5 points) Find the maximum point and the corresponding maximum objective function value if the objective is to maximize $x - y$.

(d). (5 points) Find the change in the maximum objective function value if the constraint $x \le 5$ is changed to $x \le 6$.

(e). (5 points) Find the change in the maximum objective function value if the constraint $x - y \ge 0$ is changed to $-x + y \ge 0$ while the constraint $x \le 5$ remains unchanged.

3. Consider a linear world that can be represented by a line segment [0, 10]. There are 11 persons living in this linear world, each at one integer point. Consequently, there is a person living at point x for $x = 0, 1, \ldots, 10$. Every day the 11 persons need to shop once in a supermarket.

(a). (3 points) Suppose that there is only one supermarket in this linear world, and that the supermarket is located at $x = 5$. Find the total traveling distance in one day of the 11 persons to go shopping and return back home.

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(b). (3 points) Suppose that there is only one supermarket in this linear world, and that the supermarket is located at $x = 0$. Find the total traveling distance in one day of the 11 persons to go shopping and return back home.

(c). (2 points) Compare the results from part (a) and (b). Which is a better location to locate the supermarket, $x = 0$ or 5? Give your reasons.

(d). There is already one supermarket at $x = 5$ and you would like to add new supermarket(s) to compete. Suppose that everyone goes to the closest supermarket to shop, and that each location can only accommodate at most one supermarket.

(i). (4 points) Suppose that you would only add one supermarket. Where will you locate it to maximize your market share?

(ii). (4 points) Suppose that you would add two supermarkets. Where will you locate them to maximize your market share?

(iii). (4 points) Suppose that this linear world can hold at most three supermarkets. Given the analysis in parts (i) and (ii), where is the best location for the first supermarket?

4. The following table gives the data for a balanced transportation problem. The 9 numbers encircled by the bolded line segments are the unit costs of sending an item from supplier i to customer j, $i, j \in \{1, 2, 3\}$; e.g., the cost to send 1 item from supplier 2 to customer 3, $c_{23} = 8$.

		Customer			
		1	2	3	Supply
Supplier	1	7	4	2	20
	2	5	3	8	10
	3	6	9	1	15
	Demand	10	18	17	

(a). (8 points) Put down the formulation that minimizes the total cost to ship the supplies to the customers.

(b). (3 points) Give a feasible solution for the problem.

(c). Add linear constraints to your formulation in part (a) to handle the following new situations. Answer the parts independently from each other.

(i). (3 points) Supplier 1 cannot send any item to customer 1.

(ii). (3 points) Supplier 2 should send no less items to customer 2 than to customer 3.

(iii). (3 points) The supply from supplier 3 should be shared evenly among the three customers.

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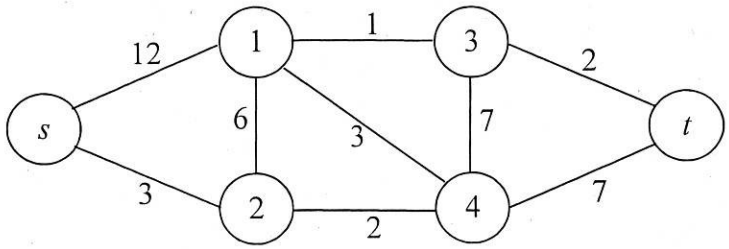
5. The following parts are related to finding the shortest path from node s to node t.



(a). (10 points) Find the shortest distance from node s to node t. Show the intermediate steps and explain the reasons.

(b). (10 points) Formulate the problem as a linear program.

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附錄

附錄列出部份題目之中譯，但一切以英文題目為本

1. 求解以下最小化問題：
(問題列於原英文題中。)

2. 一線性規劃問題之限制式為：$x - y \geq 0, x \leq 5, x \geq 0, y \geq 0$.
(a). 劃出線性規劃的可行域。
(b). 可行域有三個角點，找出其坐標。
(c). 當目標為最大化 $x - y$ 時，找出最大點和對應的最大目標函數值。
(d). 當限制式 $x \leq 5$ 變成 $x \leq 6$ 時，目標函數最大值改變了多少？
(e). 要是限制式 $x - y \geq 0$ 變成 $-x + y \geq 0$ 而限制式 $x \leq 5$ 維持不變，目標函數最大值改變了多少？

3. 問題發生在一個以線段[0，10]代表的線性世界裡。這線性世界住了 11 個人，每人的住址都是一個整數點，分別為點 x，$x = 0$，1，...，10。每天這 11 人都需要到超市購物一次。
(a). 假設這線性世界只有一家超市在 $x = 5$ 這點上。每一天這 11 人往返超市一共走了多遠？

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(b). 假設這線性世界只有一家超市在 $x = 0$ 這點上。每一天這 11 人往返超市一共走了多遠？

(c). 比較(a) 和(b) 兩子題的答案。只有一家超市時，超市設在 $x = 0$ 還是 $x = 5$ 較好？解釋所選擇的原因。

(d). 這線性世界已有一家超市設於 $x = 5$。你計劃多開些超市與現有的超市爭生意。假設每個人都選最近的超市，而每點只能容納一家超市。

(i). 要是你只多開一家超市，你會開在哪裡？

(ii). 要是你只多開兩家超市，你會開在哪裡？

(iii). 要是這線性世界最多只能容納三家超市，第一家超市開在哪裡最好？

4. 下列表格是一個平衡運輸問題的數據，給粗線段圍著的 9 個數字是從供應商 i 送一件產品到客戶 j 的單位成本，例如，從供應商 2 送一件產品到客戶 3 的價錢 $c_{23} = 8$。(表格列於原英文題中。)

(a). 列出一個規劃模型，找尋上述運輸問題最便宜的運送方式。

(b). 列出一個可行的運送方法。

(c). 在子題(a)的規劃模型多加些線性限制式以應付下列的新情況。以下子題獨自求解，不用考慮其他子題。

(i) 供應商一不能送任何產品給客戶一。

(ii) 供應商二送到客戶二的產品不比送到客戶三的少。

(iii) 供應商三的產品要均分給三個客戶

5. 以下的子題是關於找出節點 s 到節點 t 的最短路徑。(網路圖列於原英文題中。)

(a). 找出從節點 s 到節點 t 的最短矩離，列出求解的步驟，解釋步驟的理由。

(b). 寫出上述最短路徑問題的線性規劃模型。

--- 附錄完 ---