

國立交通大學 100 學年度碩士班考試入學試題

科目：機率論(4082)

考試日期：100 年 2 月 18 日 第 2 節

系所班別：統計學研究所 組別：統計所

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【不可使用計算機】*作答前請先核對試題、答案卷(試卷)與准考證之所組別與考科是否相符!!

1. You ask your neighbor to water a sickly plant while you are on vacation. Without water it will die with probability 0.8; with water it will die with probability 0.15. You are 90% certain that your neighbor will remember to water the plant.

- 10% (a) What is the probability that the plant will be alive when you return?
10% (b) If it is dead, what is the probability your neighbor forgot to water it?

2. Suppose that $P(X = a) = p$ and $P(X = b) = 1 - p$.

- 10% (a) Find a transformation of X to obtain a Bernoulli random variable.
10% (b) Find $\text{Var}\left(\frac{X}{\sqrt{p(1-p)}}\right)$.

- 20% 3. A standard Cauchy random variable X has density function

$$f(x) = \frac{1}{\pi(1+x^2)} \quad -\infty < x < \infty.$$

Show that $\frac{1}{X}$ is also a standard Cauchy random variable.

4. Let X_1, X_2, \dots be i.i.d. continuous random variables. Find

10% (a) $P(X_6 > X_1 | X_1 = \max(X_1, \dots, X_5))$

10% (b) $P(X_6 > X_2 | X_1 = \max(X_1, \dots, X_5))$

5. Let Y be a uniform random variable on $(0, 1)$. Suppose that conditional on $Y = p$, the random variable X has a binomial distribution with parameters n and p .

10% (a) Find the moment generating function of X .

10% (b) What is the distribution of X ?