Show all your work and carefully justify all your answers. Answers without explanation will not receive any score.

1. (10 points) The region below the graph of $y = \sin x$, above the x-axis and between 0 and π is rotated about the line $y = 1$. Find the volume of the solid that is generated.

2. Consider

$$\int_{-3}^{3} \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_{1}^{5-y} f(x,y,z) \mathrm{d}z\mathrm{d}y\mathrm{d}x.$$

   (a) (6 points) Describe the solid over which the integration takes place and plot its projections onto the yz-plane and xz-plane.

   (b) (8 points) Change the order of integration to $\mathrm{d}x\mathrm{d}z\mathrm{d}y$ and $\mathrm{d}y\mathrm{d}z\mathrm{d}x$.

3. (a) (7 points) Evaluate

$$\int \frac{x}{(x+1)(x+2)}\mathrm{d}x.$$

   (b) (7 points) Evaluate

$$\int_{0}^{1} \frac{\mathrm{d}x}{(2-x)\sqrt{1-x}}.$$

4. Consider the following well-known series representation of π:

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots. \qquad (1)$$

   (a) (6 points) Explain how to use (1) to estimate π to within 10^{-10}.

   (b) (6 points) If P is an estimate of π to within 10^{-n}, show that $P + \sin P$ is an estimate to within 10^{-3n}.

   Hint: Write $P = \pi + x$.

國立交通大學 100 學年度碩士班考試入學試題

科目：微積分(4051)　　　　　　　　　　　考試日期:100 年 2 月 18 日　第 4 節
系所班別：應用數學系數學建模與科學計算碩士班　　　　　第 2 頁,共 2 頁
【不可使用計算機】＊作答前請先核對試題、答案卷(試卷)與准考證之所組別與考科是否相符！！

5. (a) (6 points) Consider

$$f(x) = \int_{\tan x}^{x/4} \sin(t^2) \mathrm{d}t.$$

Compute $f'(\pi)$.

(b) (8 points) Suppose that $f(0) = 0$ and

$$f'(\ln x) = \begin{cases} 1, & \text{if } 0 < x \le 1; \\ x, & \text{if } 1 < x < \infty. \end{cases}$$

Find $f(x)$.

6. (a) (8 points) Use a suitable change of variable to compute

$$\iint_R (x+y)^2 e^{x^2-y^2} \mathrm{d}A,$$

where R is the square with vertices $(1,0), (0,1), (-1,0)$ and $(0,-1)$.

(b) (6 points) Evaluate

$$\iiint_{\mathbb{R}^3} \frac{\mathrm{d}V}{(x^2 + y^2 + z^2 + 1)^2}.$$

7. (10 points) Find absolute minimum and absolute maximum of the function
$f(x,y) = x^3 + x^2y + 2y^2$ on the domain given by $x, y \ge 0, x + y \le 1$.

8. Let $f(x) = \cos kx$ with k a positive integer.

(a) (6 points) Find

$$\frac{\mathrm{d}^n}{\mathrm{d}x^n} f(x).$$

(b) (6 points) Find all positive integers m for which $f(x)$ is a solution of

$$y'' + my = 0$$

for some value of k.